UNIVERSITY OF OULU

YLEISEN TENTIN TENTTILOMAKE - GENERAL EXAM FORM

Opiskelija täyttää / Student fills in

Opiskelijan nimi / Student name: Click here to enter text.	**Opiskelijanumero / Student number:** Click here to enter text.

Opettaja täyttää / Lecturer fills in

Opintojakson koodi / The code of the course: 721338S	
Opintojakson (tentin) nimi / The name of the course or exam: Mathematical Economics, Summer exam	
Opintopistemäärä / Credit units: 6 cr Mikäli kyseessä on välikoe, opintopistemääräksi täytetään 0 op. 0 ECTS Credits is used for mid-term exams.	
Tiedekunta / Faculty: Oulu Business School	
Tentin pvm / Date of exam: 12.8.2019	**Tentin kesto tunteina / Exam in hours: 3 h**
Tentaattori(t) / Examiner(s): Tomi Alaste	**Sisäinen postiosoite / Internal address:** 6 OYKKK
Tentissä sallitut apuvälineet / The devices allowed in the exam:	
☐ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator ☐ Muu tentissä sallittu materiaali tai apuvälineet. Tarkenna alla. / Other material or devices, allowed in the exam. Specify below. Click here to enter text. ☒ Tentissä ei ole sallittua käyttää apuvälineitä / The devices are not allowed in the exam	
Muut tenttiä koskevat ohjeet opiskelijalle (esimerkiksi kuinka moneen kysymyksen opiskelijan tulee vastata) / Other instructions for students e.g. how many questions he/she should answer: Answer all the questions	

721338S MATHEMATICAL ECONOMICS
Final exam
12.8.2019

1. It is summertime, and there are mosquitos and spiders in a yard (and no other creatures). Let us assume that each mosquito has one head and six legs, and each spider has one head and eight legs. Altogether, there are 20 heads and 140 legs in the yard.

   (a) Let x be the number of mosquitos and y the number of spiders. Write down two equations, where the first equation describes a relationship between x and y and the number of heads in the yard, and the second equation describes a relationship between x and y and the number of legs in the yard. (5 p.)

   (b) Write this system of equations in matrix form $Ax = b$. (5 p.)

   (c) Is A invertible? (*Here, you do not need to find the inverse matrix if it exists.*) (5 p.)

   (d) Solve x and y. (5 p.)

2. Consider the following national-income model

$$\begin{cases} Y = C + I_0 + G_0, \\ C = \alpha + \beta(Y - T), \\ T = \gamma + \delta Y, \end{cases}$$

   where Y is national income, C is consumption, I_0 is investment, G_0 is government expenditure, T is taxes, and $\alpha, \gamma > 0$ and $0 < \beta, \delta < 1$ are parameters.

   (a) Find the equilibrium income Y^* in terms of I_0, G_0, and the parameters. (*Hint: substitute the last equation into the second and, then, the second equation into the first one.*) (10 p.)

   (b) What happens to the equilibrium income Y^* when G_0 *increases*? (10 p.)

3. Consider a simple two-period model where a consumer's utility function is given by

$$U(x, y) = xy.$$

   Here, x and y are consumptions in periods 1 and 2, respectively. The consumer has a budget B and faces a market interest rate r, and so his/her intertemporal budget constraint is given by

$$x + \frac{y}{1+r} = B.$$

   The purpose is to find a utility maximing bundle (x^*, y^*).

   (a) Form the Lagrangian and find the critical points. (10 p.)

   (b) Form the bordered Hessian at the critical point(s). (*There is no need to qualify them.*) (5 p.)

   (c) Explain shortly what is the interpretation of the Lagrange multiplier λ in this exercise. (5 p.)

4. Consider the following equation

$$x^2 + y^2 + z^2 + 2xy + 3yz = 11.$$

   (a) Let $x = 0$ and $y = 2$. Find a *positive* z^* such that the above equation holds. (5 p.)

   (b) Show that x is a function of y and z in some neighbourhood of the point $(0, 2, z^*)$. (10 p.)

   (c) Find $\frac{\partial x}{\partial y}$ and $\frac{\partial x}{\partial z}$ at the point $(0, 2, z^*)$. (5 p.)

5. (a) Solve the differential equation $y'(t) = -2t$ under the initial condition $y(0) = 1$. (10 p.)

   (b) Consider the following discrete time model of supply (Q_t^s) and demand (Q_t^d), where P_t denotes the price at time t:

$$\begin{cases} Q_t^d = Q_t^s, \\ Q_t^d = 20 - 4P_t, \\ Q_t^s = -4 + 2P_{t-1}. \end{cases}$$

   Solve the price P_t as a function of t when $P_0 = 4$. (10 p.)