**FINANCIAL RISK MANAGEMENT** **23.4.2015**

Name: ______________________________

Student number: ______________________________

Allowed material: calculator

1. The current asset price is €5.00, and the volatility of the asset price is 25%. The risk-free rate is flat 3.0%. Consider an exotic forward contract, which pays off a euro-amount of $\sqrt{S} - K$ at the maturity of the contract, six months from now. The contract is being launched today, with an initial value of zero. All rates are in terms of continuous compounding.

   – Determine the delivery price $K$ of the contract,
   – Determine the delta of the contract,
   – Determine the gamma of the contract,

2. The current futures price of an asset is €10.00, the volatility of the asset price is 30%, and the continuously compounded dividend yield of the asset is 3%. Consider a European call option on a futures contract on the asset, with the strike price of €10.00 and the time-to-maturity of three months. The risk-free rate can be considered to be zero.

- Determine the value of the option,
- Determine the delta of the option,
- Determine the gamma of the option.

3. The tables below provide the position in sold over-the-counter options on an asset and an exchange-traded option on the same asset. You are supposed to neutralize your total position with respect to both delta and gamma on day zero.

- Determine the number of hedge options needed,
- Determine the number of underlying asset shares needed.

| Sold options | | | Initial values | | |
|---|---|---|---|---|---|
| Number of contracts | Type | Strike price | Time-to-maturity | Price | Delta |
| $-2$ | Call | 52 | $10/250$ | 0.8503 | 0.3353 |
| $-6$ | Call | 50 | $15/250$ | 2.0265 | 0.5317 |
| $-3$ | Put | 44 | $50/250$ | 1.0543 | $-0.1949$ |

| Hedge option | | | Initial values | | | |
|---|---|---|---|---|---|---|
| Number of contracts | Type | Strike price | Time-to-maturity | Price | Delta | Gamma |
| $n_h$ | Put | 50 | $40/250$ | 2.9802 | $-0.4483$ | 0.0494 |

4. The table below provides the payoff structure of an option. The current price of the underlying asset is €15, the volatility of the asset is 40%, the risk-free rate is 5%, and the time-to-maturity of the contract is six months. You may consider the structure as a modification of the Black-Scholes model or as a combination of plain vanilla options.

- Determine the price of the option,
- Determine the delta of the option,
- Determine the gamma of the option.

| Asset price | Payoff |
|---|---|
| $S_T < 10$ | 0 |
| $10 \leq S_T < 20$ | $S_T - 10$ |
| $20 \leq S_T < 30$ | 10 |
| $S_T \geq 30$ | $S_T - 20$ |

5. You are holding a €450’000 stock portfolio with a beta of 0.90 against the EURO STOXX 50 Index. You decide to adjust the beta of your portfolio to a level of 1.30 by trading Eurex June 2015 EURO STOXX 50 Index Futures. An index futures contract is on €10 per index point, and the contract currently trades at 3595.00.

   - Detemine the number of contracts traded,
   - Are you buying or selling the contracts?

**Asset price process:**

(1) $$dS = (\mu - q)Sdt + \sigma S dz$$

(2) $$df = \left( \frac{\partial f}{\partial S}(\mu - q)S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S}\sigma S dz$$

(3) $$E_t(S_T) = S_t e^{(\mu-q)(T-t)}$$

(4) $$Std_t(S_T) = S_t e^{(\mu-q)(T-t)} \sqrt{e^{\sigma^2(T-t)} - 1}$$

(5) $$m = E_t(\ln S_T) = \ln S_t + \left(\mu - q - \frac{\sigma^2}{2}\right)(T - t)$$

(6) $$s = Std_t(\ln S_T) = \sigma\sqrt{T - t}$$

**Notice:**

In the case of a non-dividend-paying asset, ignore the dividend yield $q$.

In the case of an exchange rate, replace $\mu$ with the risk-free rate $r$, and $q$ with the foreign currency risk-free rate $r_f$.

In the case of a forward/futures price, replace $q$ with the risk-free rate $r$.

In the case of a risk-neutral process, replace $\mu$ with the risk-free rate $r$.

**Forward contract**

(1) $$c_t = e^{-r(T-t)}\hat{E}[S_T - K] = e^{-r(T-t)} \int_{-\infty}^{\infty} (S_T - K)\, \hat{p}(S_T)\, dS_T$$

(2) $$= e^{-r(T-t)} \left[ \int_{-\infty}^{\infty} S_T\, \hat{p}(S_T)\, dS_T - K \int_{-\infty}^{\infty} \hat{p}(S_T)\, dS_T \right]$$

(3) $$= e^{-r(T-t)} \left[ \hat{E}(S_T) - K \right]$$

(4) $$= e^{-r(T-t)} \left[ F_t - K \right]$$

(5) $$= e^{-r(T-t)} \left[ S_0 e^{r(T-t)} - K \right]$$

(6) $$= S_0 - K e^{-r(T-t)}$$

**European call option:**

(1) $$c_t = e^{-r(T-t)}\hat{E}[max(0, S_T - X)] = e^{-r(T-t)}\int_X^{\infty}(S_T - X)\,\hat{p}(S_T)\,dS_T$$

(2) $$= e^{-r(T-t)}\left[\int_X^{\infty} S_T\,\hat{p}(S_T)\,dS_T - X\int_X^{\infty}\hat{p}(S_T)\,dS_T\right]$$

(3) $$= e^{-r(T-t)}\left[\int_X^{\infty} S_T\,\frac{1}{s\,S_T\sqrt{2\pi}}e^{\frac{-(\ln S_T-\hat{m})^2}{2s^2}}\,dS_T - X\int_X^{\infty}\frac{1}{s\,S_T\sqrt{2\pi}}e^{\frac{-(\ln S_T-\hat{m})^2}{2s^2}}\,dS_T\right]$$

(4) $$= e^{-r(T-t)}\left[\hat{E}(S_T)N\left(\frac{\hat{m}-\ln X+s^2}{s}\right) - XN\left(\frac{\hat{m}-\ln X}{s}\right)\right]$$

(5) $$= S_te^{-q(T-t)}N\left(\frac{\ln(S_t/X)+(r-q+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}\right) - Xe^{-r(T-t)}N\left(\frac{\ln(S_t/X)+(r-q-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}\right)$$

**European put option:**

(1) $$p_t = e^{-r(T-t)}\hat{E}[max(0, X - S_T)] = e^{-r(T-t)}\int_X^{\infty}(X - S_T)\,\hat{p}(S_T)\,dS_T$$

(2) $$= e^{-r(T-t)}\left[X\int_{-\infty}^{X}\hat{p}(S_T)\,dS_T - \int_{-\infty}^{X} S_T\,\hat{p}(S_T)\,dS_T\right]$$

(3) $$= e^{-r(T-t)}\left[X\int_{-\infty}^{X}\frac{1}{s\,S_T\sqrt{2\pi}}e^{\frac{-(\ln S_T-\hat{m})^2}{2s^2}}\,dS_T - \int_{-\infty}^{X} S_T\,\frac{1}{s\,S_T\sqrt{2\pi}}e^{\frac{-(\ln S_T-\hat{m})^2}{2s^2}}\,dS_T\right]$$

(4) $$= e^{-r(T-t)}\left[XN\left(-\frac{\hat{m}-\ln X}{s}\right)\right] - \hat{E}(S_T)N\left(-\frac{\hat{m}-\ln X+s^2}{s}\right)\Bigg]$$

(5) $$= Xe^{-r(T-t)}N\left(-\frac{\ln(S_t/X)+(r-q-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}\right) - S_te^{-q(T-t)}N\left(-\frac{\ln(S_t/X)+(r-q+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}\right)$$

**Notice:**

Equations (3) and (4) assume lognormally distiributed price at time $T$.
– $\hat{m} = \hat{E}(\ln S_T)$, $s = Std(\ln S_T)$

Equation (5) assumes geometric Brownian motion of asset price.
– $\hat{m} = \hat{E}(\ln S_T) = \ln S_t + (r - q - \sigma^2/2)(T-t)$, $s = Std(\ln S_T) = \sigma\sqrt{T-t}$

Currency options: Apply (5), replace $q$ with the foreign currency risk-free rate $r_f$.

Futures options: Apply (5), replace $S_t$ with the futures price $F_t$, and $q$ with the risk-free rate $r$.

Bond options: Apply (5), replace $S_t$ with the forward price $F_t$ of the bond, $q$ with the risk-free rate $r$, and $\sigma$ with the volatility rate $\hat{\dot{\sigma}} = \dot{s}/\sqrt{T-t}$, where $\dot{s}$ is the standard deviation of $\ln F_T$.

**Delta of a European call option:**

$$(1) \qquad c_t = S_t e^{-q(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2)$$

$$(2) \qquad \frac{\partial c_t}{\partial S_t} = e^{-q(T-t)}\, N(d_1) + S_t e^{-q(T-t)} N'(d_1)\, \frac{\partial d_1}{\partial S_t} - X e^{-r(T-t)} N'(d_2)\, \frac{\partial d_2}{\partial S_t}$$

$$(3) \qquad = e^{-q(T-t)}\, N(d_1) + S_t e^{-q(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}} - X e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_2^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}}$$

$$(4) \qquad = e^{-q(T-t)}\, N(d_1)$$

**Delta of a European put option:**

$$(1) \qquad p_t = X e^{-r(T-t)} N(-d_2) - S_t e^{-q(T-t)} N(-d_1)$$

$$(2) \qquad \frac{\partial p_t}{\partial S_t} = X e^{-r(T-t)} N'(-d_2)\, \frac{\partial(-d_2)}{\partial S_t} - e^{-q(T-t)}\, N(-d_1) + S_t e^{-q(T-t)} N'(d_1)\, \frac{\partial(-d_1)}{\partial S_t}$$

$$(3) \qquad = -X e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_2^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}} - e^{-q(T-t)}\, N(-d_1) - S_t e^{-q(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}}$$

$$(4) \qquad = e^{-q(T-t)}\, [N(d_1) - 1]$$

**Delta and gamma of a European call option and a European put option:**

$$(5) \qquad \frac{\partial^2 c_t}{\partial S_t^2} = \frac{\partial^2 p_t}{\partial S_t^2} = e^{-q(T-t)}\, N'(d_1)\, \frac{\partial d_1}{\partial S_t}$$

$$(6) \qquad = e^{-q(T-t)}\, \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}}$$

**Notice:**

Equation (3) is a general result, which holds true, even if the trigger price levels $X$ in $d_1$, and $d_2$, and the strike price $X$ in the main equation, differ from each other.

**Some basic calculus:**

$\ln xy = \ln x + \ln y$ $\qquad \ln \frac{x}{y} = \ln x - \ln y$ $\qquad e^{f(x)} e^{g(x)} = e^{f(x)+g(x)}$

$$D\Big[f(x)g(x)\Big] = f'(x)g(x) + f(x)g'(x)$$

$$D\Big[f[g(x)]\Big] = f'[g(x)]g'(x)$$

$D\Big[f(x)^n\Big] = n f(x)^{n-1} f'(x)$ $\qquad D\Big[e^{f(x)}\Big] = f'(x) e^{f(x)}$ $\qquad D\Big[\ln f(x)\Big] = f'(x) \frac{1}{f(x)}$

| | 00 | 05 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| −2.4 | 0.0082 | 0.0081 | 0.0080 | 0.0079 | 0.0078 | 0.0077 | 0.0075 | 0.0074 | 0.0073 | 0.0072 | 0.0071 | 0.0070 | 0.0069 | 0.0069 | 0.0068 | 0.0067 | 0.0066 | 0.0065 | 0.0064 | 0.0063 |
| −2.3 | 0.0107 | 0.0106 | 0.0104 | 0.0103 | 0.0102 | 0.0100 | 0.0099 | 0.0098 | 0.0096 | 0.0095 | 0.0094 | 0.0093 | 0.0091 | 0.0090 | 0.0089 | 0.0088 | 0.0087 | 0.0085 | 0.0084 | 0.0083 |
| −2.2 | 0.0139 | 0.0137 | 0.0136 | 0.0134 | 0.0132 | 0.0130 | 0.0129 | 0.0127 | 0.0125 | 0.0124 | 0.0122 | 0.0121 | 0.0119 | 0.0118 | 0.0116 | 0.0115 | 0.0113 | 0.0112 | 0.0110 | 0.0109 |
| −2.1 | 0.0179 | 0.0176 | 0.0174 | 0.0172 | 0.0170 | 0.0168 | 0.0166 | 0.0164 | 0.0162 | 0.0160 | 0.0158 | 0.0156 | 0.0154 | 0.0152 | 0.0150 | 0.0148 | 0.0146 | 0.0144 | 0.0143 | 0.0141 |
| −2.0 | 0.0228 | 0.0225 | 0.0222 | 0.0220 | 0.0217 | 0.0214 | 0.0212 | 0.0209 | 0.0207 | 0.0204 | 0.0202 | 0.0199 | 0.0197 | 0.0195 | 0.0192 | 0.0190 | 0.0188 | 0.0185 | 0.0183 | 0.0181 |
| −1.9 | 0.0287 | 0.0284 | 0.0281 | 0.0277 | 0.0274 | 0.0271 | 0.0268 | 0.0265 | 0.0262 | 0.0259 | 0.0256 | 0.0253 | 0.0250 | 0.0247 | 0.0244 | 0.0241 | 0.0239 | 0.0236 | 0.0233 | 0.0230 |
| −1.8 | 0.0359 | 0.0355 | 0.0351 | 0.0348 | 0.0344 | 0.0340 | 0.0336 | 0.0333 | 0.0329 | 0.0325 | 0.0322 | 0.0318 | 0.0314 | 0.0311 | 0.0307 | 0.0304 | 0.0301 | 0.0297 | 0.0294 | 0.0290 |
| −1.7 | 0.0446 | 0.0441 | 0.0436 | 0.0432 | 0.0427 | 0.0423 | 0.0418 | 0.0414 | 0.0409 | 0.0405 | 0.0401 | 0.0396 | 0.0392 | 0.0388 | 0.0384 | 0.0379 | 0.0375 | 0.0371 | 0.0367 | 0.0363 |
| −1.6 | 0.0548 | 0.0542 | 0.0537 | 0.0532 | 0.0526 | 0.0521 | 0.0516 | 0.0510 | 0.0505 | 0.0500 | 0.0495 | 0.0490 | 0.0485 | 0.0480 | 0.0475 | 0.0470 | 0.0465 | 0.0460 | 0.0455 | 0.0450 |
| −1.5 | 0.0668 | 0.0662 | 0.0655 | 0.0649 | 0.0643 | 0.0636 | 0.0630 | 0.0624 | 0.0618 | 0.0612 | 0.0606 | 0.0600 | 0.0594 | 0.0588 | 0.0582 | 0.0576 | 0.0571 | 0.0565 | 0.0559 | 0.0554 |
| −1.4 | 0.0808 | 0.0800 | 0.0793 | 0.0785 | 0.0778 | 0.0771 | 0.0764 | 0.0756 | 0.0749 | 0.0742 | 0.0735 | 0.0728 | 0.0721 | 0.0715 | 0.0708 | 0.0701 | 0.0694 | 0.0688 | 0.0681 | 0.0675 |
| −1.3 | 0.0968 | 0.0959 | 0.0951 | 0.0943 | 0.0934 | 0.0926 | 0.0918 | 0.0909 | 0.0901 | 0.0893 | 0.0885 | 0.0877 | 0.0869 | 0.0861 | 0.0853 | 0.0846 | 0.0838 | 0.0830 | 0.0823 | 0.0815 |
| −1.2 | 0.1151 | 0.1141 | 0.1131 | 0.1122 | 0.1112 | 0.1103 | 0.1093 | 0.1084 | 0.1075 | 0.1066 | 0.1056 | 0.1047 | 0.1038 | 0.1029 | 0.1020 | 0.1012 | 0.1003 | 0.0994 | 0.0985 | 0.0977 |
| −1.1 | 0.1357 | 0.1346 | 0.1335 | 0.1324 | 0.1314 | 0.1303 | 0.1292 | 0.1282 | 0.1271 | 0.1261 | 0.1251 | 0.1240 | 0.1230 | 0.1220 | 0.1210 | 0.1200 | 0.1190 | 0.1180 | 0.1170 | 0.1160 |
| −1.0 | 0.1587 | 0.1574 | 0.1562 | 0.1551 | 0.1539 | 0.1527 | 0.1515 | 0.1503 | 0.1492 | 0.1480 | 0.1469 | 0.1457 | 0.1446 | 0.1434 | 0.1423 | 0.1412 | 0.1401 | 0.1390 | 0.1379 | 0.1368 |
| −0.9 | 0.1841 | 0.1827 | 0.1814 | 0.1801 | 0.1788 | 0.1775 | 0.1762 | 0.1749 | 0.1736 | 0.1723 | 0.1711 | 0.1698 | 0.1685 | 0.1673 | 0.1660 | 0.1648 | 0.1635 | 0.1623 | 0.1611 | 0.1599 |
| −0.8 | 0.2119 | 0.2104 | 0.2090 | 0.2075 | 0.2061 | 0.2047 | 0.2033 | 0.2019 | 0.2005 | 0.1991 | 0.1977 | 0.1963 | 0.1949 | 0.1935 | 0.1922 | 0.1908 | 0.1894 | 0.1881 | 0.1867 | 0.1854 |
| −0.7 | 0.2420 | 0.2404 | 0.2389 | 0.2373 | 0.2358 | 0.2342 | 0.2327 | 0.2312 | 0.2296 | 0.2281 | 0.2266 | 0.2251 | 0.2236 | 0.2221 | 0.2206 | 0.2192 | 0.2177 | 0.2162 | 0.2148 | 0.2133 |
| −0.6 | 0.2743 | 0.2726 | 0.2709 | 0.2693 | 0.2676 | 0.2660 | 0.2643 | 0.2627 | 0.2611 | 0.2595 | 0.2578 | 0.2562 | 0.2546 | 0.2530 | 0.2514 | 0.2498 | 0.2483 | 0.2467 | 0.2451 | 0.2435 |
| −0.5 | 0.3085 | 0.3068 | 0.3050 | 0.3033 | 0.3015 | 0.2998 | 0.2981 | 0.2963 | 0.2946 | 0.2929 | 0.2912 | 0.2894 | 0.2877 | 0.2860 | 0.2843 | 0.2826 | 0.2810 | 0.2793 | 0.2776 | 0.2759 |
| −0.4 | 0.3446 | 0.3427 | 0.3409 | 0.3391 | 0.3372 | 0.3354 | 0.3336 | 0.3318 | 0.3300 | 0.3282 | 0.3264 | 0.3246 | 0.3228 | 0.3210 | 0.3192 | 0.3174 | 0.3156 | 0.3138 | 0.3121 | 0.3103 |
| −0.3 | 0.3821 | 0.3802 | 0.3783 | 0.3764 | 0.3745 | 0.3726 | 0.3707 | 0.3688 | 0.3669 | 0.3650 | 0.3632 | 0.3613 | 0.3594 | 0.3576 | 0.3557 | 0.3538 | 0.3520 | 0.3501 | 0.3483 | 0.3464 |
| −0.2 | 0.4207 | 0.4188 | 0.4168 | 0.4149 | 0.4129 | 0.4110 | 0.4090 | 0.4071 | 0.4052 | 0.4032 | 0.4013 | 0.3994 | 0.3974 | 0.3955 | 0.3936 | 0.3917 | 0.3897 | 0.3878 | 0.3859 | 0.3840 |
| −0.1 | 0.4602 | 0.4582 | 0.4562 | 0.4542 | 0.4522 | 0.4503 | 0.4483 | 0.4463 | 0.4443 | 0.4424 | 0.4404 | 0.4384 | 0.4364 | 0.4345 | 0.4325 | 0.4305 | 0.4286 | 0.4266 | 0.4247 | 0.4227 |
| −0.0 | 0.5000 | 0.4980 | 0.4960 | 0.4940 | 0.4920 | 0.4900 | 0.4880 | 0.4860 | 0.4840 | 0.4821 | 0.4801 | 0.4781 | 0.4761 | 0.4741 | 0.4721 | 0.4701 | 0.4681 | 0.4661 | 0.4641 | 0.4622 |
| 0.0 | 0.5000 | 0.5020 | 0.5040 | 0.5060 | 0.5080 | 0.5100 | 0.5120 | 0.5140 | 0.5160 | 0.5179 | 0.5199 | 0.5219 | 0.5239 | 0.5259 | 0.5279 | 0.5299 | 0.5319 | 0.5339 | 0.5359 | 0.5378 |
| 0.1 | 0.5398 | 0.5418 | 0.5438 | 0.5458 | 0.5478 | 0.5497 | 0.5517 | 0.5537 | 0.5557 | 0.5576 | 0.5596 | 0.5616 | 0.5636 | 0.5655 | 0.5675 | 0.5695 | 0.5714 | 0.5734 | 0.5753 | 0.5773 |
| 0.2 | 0.5793 | 0.5812 | 0.5832 | 0.5851 | 0.5871 | 0.5890 | 0.5910 | 0.5929 | 0.5948 | 0.5968 | 0.5987 | 0.6006 | 0.6026 | 0.6045 | 0.6064 | 0.6083 | 0.6103 | 0.6122 | 0.6141 | 0.6160 |
| 0.3 | 0.6179 | 0.6198 | 0.6217 | 0.6236 | 0.6255 | 0.6274 | 0.6293 | 0.6312 | 0.6331 | 0.6350 | 0.6368 | 0.6387 | 0.6406 | 0.6424 | 0.6443 | 0.6462 | 0.6480 | 0.6499 | 0.6517 | 0.6536 |
| 0.4 | 0.6554 | 0.6573 | 0.6591 | 0.6609 | 0.6628 | 0.6646 | 0.6664 | 0.6682 | 0.6700 | 0.6718 | 0.6736 | 0.6754 | 0.6772 | 0.6790 | 0.6808 | 0.6826 | 0.6844 | 0.6862 | 0.6879 | 0.6897 |
| 0.5 | 0.6915 | 0.6932 | 0.6950 | 0.6967 | 0.6985 | 0.7002 | 0.7019 | 0.7037 | 0.7054 | 0.7071 | 0.7088 | 0.7106 | 0.7123 | 0.7140 | 0.7157 | 0.7174 | 0.7190 | 0.7207 | 0.7224 | 0.7241 |
| 0.6 | 0.7257 | 0.7274 | 0.7291 | 0.7307 | 0.7324 | 0.7340 | 0.7357 | 0.7373 | 0.7389 | 0.7405 | 0.7422 | 0.7438 | 0.7454 | 0.7470 | 0.7486 | 0.7502 | 0.7517 | 0.7533 | 0.7549 | 0.7565 |
| 0.7 | 0.7580 | 0.7596 | 0.7611 | 0.7627 | 0.7642 | 0.7658 | 0.7673 | 0.7688 | 0.7704 | 0.7719 | 0.7734 | 0.7749 | 0.7764 | 0.7779 | 0.7794 | 0.7808 | 0.7823 | 0.7838 | 0.7852 | 0.7867 |
| 0.8 | 0.7881 | 0.7896 | 0.7910 | 0.7925 | 0.7939 | 0.7953 | 0.7967 | 0.7981 | 0.7995 | 0.8009 | 0.8023 | 0.8037 | 0.8051 | 0.8065 | 0.8078 | 0.8092 | 0.8106 | 0.8119 | 0.8133 | 0.8146 |
| 0.9 | 0.8159 | 0.8173 | 0.8186 | 0.8199 | 0.8212 | 0.8225 | 0.8238 | 0.8251 | 0.8264 | 0.8277 | 0.8289 | 0.8302 | 0.8315 | 0.8327 | 0.8340 | 0.8352 | 0.8365 | 0.8377 | 0.8389 | 0.8401 |
| 1.0 | 0.8413 | 0.8426 | 0.8438 | 0.8449 | 0.8461 | 0.8473 | 0.8485 | 0.8497 | 0.8508 | 0.8520 | 0.8531 | 0.8543 | 0.8554 | 0.8566 | 0.8577 | 0.8588 | 0.8599 | 0.8610 | 0.8621 | 0.8632 |
| 1.1 | 0.8643 | 0.8654 | 0.8665 | 0.8676 | 0.8686 | 0.8697 | 0.8708 | 0.8718 | 0.8729 | 0.8739 | 0.8749 | 0.8760 | 0.8770 | 0.8780 | 0.8790 | 0.8800 | 0.8810 | 0.8820 | 0.8830 | 0.8840 |
| 1.2 | 0.8849 | 0.8859 | 0.8869 | 0.8878 | 0.8888 | 0.8897 | 0.8907 | 0.8916 | 0.8925 | 0.8934 | 0.8944 | 0.8953 | 0.8962 | 0.8971 | 0.8980 | 0.8988 | 0.8997 | 0.9006 | 0.9015 | 0.9023 |
| 1.3 | 0.9032 | 0.9041 | 0.9049 | 0.9057 | 0.9066 | 0.9074 | 0.9082 | 0.9091 | 0.9099 | 0.9107 | 0.9115 | 0.9123 | 0.9131 | 0.9139 | 0.9147 | 0.9154 | 0.9162 | 0.9170 | 0.9177 | 0.9185 |
| 1.4 | 0.9192 | 0.9200 | 0.9207 | 0.9215 | 0.9222 | 0.9229 | 0.9236 | 0.9244 | 0.9251 | 0.9258 | 0.9265 | 0.9272 | 0.9279 | 0.9285 | 0.9292 | 0.9299 | 0.9306 | 0.9312 | 0.9319 | 0.9325 |
| 1.5 | 0.9332 | 0.9338 | 0.9345 | 0.9351 | 0.9357 | 0.9364 | 0.9370 | 0.9376 | 0.9382 | 0.9388 | 0.9394 | 0.9400 | 0.9406 | 0.9412 | 0.9418 | 0.9424 | 0.9429 | 0.9435 | 0.9441 | 0.9446 |
| 1.6 | 0.9452 | 0.9458 | 0.9463 | 0.9468 | 0.9474 | 0.9479 | 0.9484 | 0.9490 | 0.9495 | 0.9500 | 0.9505 | 0.9510 | 0.9515 | 0.9520 | 0.9525 | 0.9530 | 0.9535 | 0.9540 | 0.9545 | 0.9550 |
| 1.7 | 0.9554 | 0.9559 | 0.9564 | 0.9568 | 0.9573 | 0.9577 | 0.9582 | 0.9586 | 0.9591 | 0.9595 | 0.9599 | 0.9604 | 0.9608 | 0.9612 | 0.9616 | 0.9621 | 0.9625 | 0.9629 | 0.9633 | 0.9637 |
| 1.8 | 0.9641 | 0.9645 | 0.9649 | 0.9652 | 0.9656 | 0.9660 | 0.9664 | 0.9667 | 0.9671 | 0.9675 | 0.9678 | 0.9682 | 0.9686 | 0.9689 | 0.9693 | 0.9696 | 0.9699 | 0.9703 | 0.9706 | 0.9710 |
| 1.9 | 0.9713 | 0.9716 | 0.9719 | 0.9723 | 0.9726 | 0.9729 | 0.9732 | 0.9735 | 0.9738 | 0.9741 | 0.9744 | 0.9747 | 0.9750 | 0.9753 | 0.9756 | 0.9759 | 0.9761 | 0.9764 | 0.9767 | 0.9770 |
| 2.0 | 0.9772 | 0.9775 | 0.9778 | 0.9780 | 0.9783 | 0.9786 | 0.9788 | 0.9791 | 0.9793 | 0.9796 | 0.9798 | 0.9801 | 0.9803 | 0.9805 | 0.9808 | 0.9810 | 0.9812 | 0.9815 | 0.9817 | 0.9819 |
| 2.1 | 0.9821 | 0.9824 | 0.9826 | 0.9828 | 0.9830 | 0.9832 | 0.9834 | 0.9836 | 0.9838 | 0.9840 | 0.9842 | 0.9844 | 0.9846 | 0.9848 | 0.9850 | 0.9852 | 0.9854 | 0.9856 | 0.9857 | 0.9859 |
| 2.2 | 0.9861 | 0.9863 | 0.9864 | 0.9866 | 0.9868 | 0.9870 | 0.9871 | 0.9873 | 0.9875 | 0.9876 | 0.9878 | 0.9879 | 0.9881 | 0.9882 | 0.9884 | 0.9885 | 0.9887 | 0.9888 | 0.9890 | 0.9891 |
| 2.3 | 0.9893 | 0.9894 | 0.9896 | 0.9897 | 0.9898 | 0.9900 | 0.9901 | 0.9902 | 0.9904 | 0.9905 | 0.9906 | 0.9907 | 0.9909 | 0.9910 | 0.9911 | 0.9912 | 0.9913 | 0.9915 | 0.9916 | 0.9917 |
| 2.4 | 0.9918 | 0.9919 | 0.9920 | 0.9921 | 0.9922 | 0.9923 | 0.9925 | 0.9926 | 0.9927 | 0.9928 | 0.9929 | 0.9930 | 0.9931 | 0.9931 | 0.9932 | 0.9933 | 0.9934 | 0.9935 | 0.9936 | 0.9937 |