LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit)
SCI, OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

UNIVERSITY OF OULU

YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

The code and the name of the course: **721960S** **Financial Risk Management**	
Oulu Business School	
26.04.2017	**3 h**
Exam 1	**6 cu**
Jukka Perttunen	

Sallitut apuvälineet / The devices allowed in the exam:

☒ Nelilaskin / Standard calculator ☒ Funktiolaskin / Scientific calculator ☒ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintojakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

☒ Kyllä / Yes ☐ Ei / No

FINANCIAL RISK MANAGEMENT **26.4.2017**

Name: ______________________________

Student number: ______________________________

Allowed material: calculator
All asnswers and solutions on these sheets only!

1. Let us assume that an asset price S follows the process

$$dS = \mu S dt + \sigma S dz.$$

Under the assumption above, the cubic S^3 of the asset price follows the process

$$dS^3 = (3\mu + 2\sigma^2)S^3 dt + 3\sigma S^3 dz.$$

Correspondingly, the expected value of the cubic of the asset price at time T in future is

$$E[S^3] = S^3 e^{(3\mu+2\sigma^2)T}.$$

Suppose that you are about to sell an exotic forward contract which pays off an amout of $S^3 - K$ at the maturity of the contract, six months from now. The current asset price is $S =$ €10, the volatility of the asset price is $\sigma = 0.30$, the expected return of the asset is $\mu = 12.8\%$, and the risk-free rate is $r = 4.00\%$.

a) Determine the delivery price K which sets the initial value of the contract equal to zero.

b) How many shares of the underlying asset is needed to delta-nautralize a short position of 250 contracts?

2. The table below provides a monthly time-series of an asset price and the month 6 futures price of the asset.

	A	B	C	D	E	F	G	H
1	Time in months	0	1	2	3	4	5	6
2	Asset price	20.00	19.20	19.80	19.25	18.90	18.80	19.00
3	Month 6 futures price	20.50	19.60	20.13	19.49	19.06	18.88	19.00
4	Margin requirement	2.00	2.00	2.00	2.00	2.00	2.00	2.00
5	Deposit by us	2.00						
6	Deposit by the exchange	0.00						
7	Margin account	2.00						

Calculate the deposits and the level of the margin account in a case of short position of one futures contract. The margining of the contract is assumed to be carried out on a monthly basis. Show the calculations below, and fill in the missing values in the table.

3. The table below provides the short positions in three options on an asset.

	A	B	C	D	E	F	G	H	I	J
1	Asset price	20								
2	Volatility (%)	40								
3	Risk-free rate (%)	3								
4									Position	
5	Option type	X	T	D1	N(D1)	Delta	Gamma	N	Delta	Gamma
6	Call	22	0.81	−0.0173	0.4931			−30		
7	Call	18	0.36	0.6040	0.7271			−24		
8	Put	16	0.64	0.9173	0.8205			−10		
9	Total									
10										
11	Put option (hedge)	20	1.00	0.2750	0.6083					
12	Asset (hedge)									
13	Total									

The table provides the strike prices X and the maturities T of the shorted options, and those of an option, which is available for the neutralizing of the position with respect to its gamma. The table also provides the number N of the contracts sold for each of the three shorted options. The values of d_1 as well as the corresponding cumulative probabilities $N(d_1)$ of the standardized normal distribution are also available:

$$d_1 = \frac{\ln(S/X) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}.$$

a) How many option contracts is needed to gamma-neutralize the shorted option position?

b) How many asset shares is needed to delta-neutralize the gamma-neutralized total position?

Calculate below, and fill in the table, the values of F6:F8, F11:F12, G6:G8, G11:G12, H11:H12, I6:I9, I11:I13, J6:J9 and J11:J13.

Calculate the values of the deltas and the gammas of the contracts with a precision of four decimals, and the delta and the gammas of the position with a precision of two decimals. Round the number of contracts to the nearest integer.

4. The table below provides the quotations of interest rate futures and options on interest rate futures.

Interest rate futures / Options on interest rate futures			
Three-month Euribor rate	1.478%		
Futures price	98.520		
Futures option quotations	Call	Strike	Put
	0.02	99.00	0.48
	0.04	98.50	0.03
	0.57	98.00	0.02
	1.04	97.50	0.01
	1.53	97.00	0.01
Contract unit is €1'000'000.			
Quotes are in percentage points.			

a) What is the payoff from a short position of 20 futures contracts, if the forward interest rate of the period in question is 1.8% at the time of the closing of the position, and the futures price is strictly in line with the quoted forward interest rate?

b) What is the bying price of 20 put options on futures with the strike of 99.000?

5. The table below provides the payoff structure of an option. The current price of the underlying asset is €20, the volatility of the asset is 40%, the risk-free rate is 3%, and the time-to-maturity of the contract is six months. You may consider the structure as a modification of the Black-Scholes model or as a combination of plain vanilla options.

Asset price	Payoff
$S_T < 15$	$15 - S_T$
$15 \leq S_T < 20$	0
$20 \leq S_T < 25$	$S_T - 20$
$S_T \geq 25$	5

a) Calculate the value of the option.

b) Calculate the delta of the option.

Asset price process:

(1) $dS = (\mu - q)Sdt + \sigma Sdz$

(2) $df = \left(\frac{\partial f}{\partial S}(\mu - q)S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S}\sigma Sdz$

(3) $E_t(S_T) = S_t e^{(\mu-q)(T-t)}$

(4) $Std_t(S_T) = S_t e^{(\mu-q)(T-t)}\sqrt{e^{\sigma^2(T-t)} - 1}$

(5) $m = E_t(\ln S_T) = \ln S_t + \left(\mu - q - \frac{\sigma^2}{2}\right)(T - t)$

(6) $s = Std_t(\ln S_T) = \sigma\sqrt{T - t}$

Notice:

In the case of a non-dividend-paying asset, ignore the dividend yield q.

In the case of an exchange rate, replace μ with the risk-free rate r, and q with the foreign currency risk-free rate r_f.

In the case of a forward/futures price, replace q with the risk-free rate r.

In the case of a risk-neutral process, replace μ with the risk-free rate r.

Forward contract

(1) $c_t = e^{-r(T-t)}\hat{E}[S_T - K] = e^{-r(T-t)}\int_{-\infty}^{\infty}(S_T - K)\,\hat{p}(S_T)\,dS_T$

(2) $= e^{-r(T-t)}\left[\int_{-\infty}^{\infty} S_T\,\hat{p}(S_T)\,dS_T - K\int_{-\infty}^{\infty}\hat{p}(S_T)\,dS_T\right]$

(3) $= e^{-r(T-t)}\left[\hat{E}(S_T) - K\right]$

(4) $= e^{-r(T-t)}\left[F_t - K\right]$

(5) $= e^{-r(T-t)}\left[S_0 e^{r(T-t)} - K\right]$

(6) $= S_0 - Ke^{-r(T-t)}$

European call option:

(1) $$c_t = e^{-r(T-t)}\hat{E}[max(0, S_T - X)] = e^{-r(T-t)} \int_X^{\infty} (S_T - X)\, \hat{p}(S_T)\, dS_T$$

(2) $$= e^{-r(T-t)} \left[\int_X^{\infty} S_T\, \hat{p}(S_T)\, dS_T - X \int_X^{\infty} \hat{p}(S_T)\, dS_T \right]$$

(3) $$= e^{-r(T-t)} \left[\int_X^{\infty} S_T \frac{1}{s\, S_T \sqrt{2\pi}} e^{\frac{-(\ln S_T - \hat{m})^2}{2s^2}}\, dS_T - X \int_X^{\infty} \frac{1}{s\, S_T\sqrt{2\pi}} e^{\frac{-(\ln S_T - \hat{m})^2}{2s^2}}\, dS_T \right]$$

(4) $$= e^{-r(T-t)} \left[\hat{E}(S_T) N\left(\frac{\hat{m} - \ln X + s^2}{s} \right) - XN\left(\frac{\hat{m} - \ln X}{s} \right) \right]$$

(5) $$= S_t e^{-q(T-t)} N\left(\frac{\ln(S_t/X) + (r - q + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \right) - Xe^{-r(T-t)} N\left(\frac{\ln(S_t/X) + (r - q - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \right)$$

European put option:

(1) $$p_t = e^{-r(T-t)}\hat{E}[max(0, X - S_T)] = e^{-r(T-t)} \int_X^{\infty} (X - S_T)\, \hat{p}(S_T)\, dS_T$$

(2) $$= e^{-r(T-t)} \left[X \int_{-\infty}^{X} \hat{p}(S_T)\, dS_T - \int_{-\infty}^{X} S_T\, \hat{p}(S_T)\, dS_T \right]$$

(3) $$= e^{-r(T-t)} \left[X \int_{-\infty}^{X} \frac{1}{s\, S_T\sqrt{2\pi}} e^{\frac{-(\ln S_T - \hat{m})^2}{2s^2}}\, dS_T - \int_{-\infty}^{X} S_T \frac{1}{s\, S_T\sqrt{2\pi}} e^{\frac{-(\ln S_T - \hat{m})^2}{2s^2}}\, dS_T \right]$$

(4) $$= e^{-r(T-t)} \left[XN\left(-\frac{\hat{m} - \ln X}{s} \right) \right] - \hat{E}(S_T) N\left(-\frac{\hat{m} - \ln X + s^2}{s} \right) \Big]$$

(5) $$= Xe^{-r(T-t)} N\left(-\frac{\ln(S_t/X) + (r - q - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \right) - S_t e^{-q(T-t)} N\left(-\frac{\ln(S_t/X) + (r - q + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \right)$$

Notice:

Equations (3) and (4) assume lognormally distiributed price at time T.
– $\hat{m} = \hat{E}(\ln S_T)$, $s = Std(\ln S_T)$

Equation (5) assumes geometric Brownian motion of asset price.
– $\hat{m} = \hat{E}(\ln S_T) = \ln S_t + (r - q - \sigma^2/2)(T-t)$, $s = Std(\ln S_T) = \sigma\sqrt{T-t}$

Currency options: Apply (5), replace q with the foreign currency risk-free rate r_f.

Futures options: Apply (5), replace S_t with the futures price F_t, and q with the risk-free rate r.

Bond options: Apply (5), replace S_t with the forward price F_t of the bond, q with the risk-free rate r, and σ with the volatility rate $\hat{\hat{\sigma}} = \dot{s}/\sqrt{T-t}$, where $\dot{s}$ is the standard deviation of $\ln F_T$.

Delta of a European call option:

(1) $c_t = S_t e^{-q(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2)$

(2) $\frac{\partial c_t}{\partial S_t} = e^{-q(T-t)}\, N(d_1) + S_t e^{-q(T-t)} N'(d_1)\, \frac{\partial d_1}{\partial S_t} - X e^{-r(T-t)} N'(d_2)\, \frac{\partial d_2}{\partial S_t}$

(3) $= e^{-q(T-t)}\, N(d_1) + S_t e^{-q(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}} - X e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{d_2^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}}$

(4) $= e^{-q(T-t)}\, N(d_1)$

Delta of a European put option:

(1) $p_t = X e^{-r(T-t)} N(-d_2) - S_t e^{-q(T-t)} N(-d_1)$

(2) $\frac{\partial p_t}{\partial S_t} = X e^{-r(T-t)} N'(-d_2)\, \frac{\partial(-d_2)}{\partial S_t} - e^{-q(T-t)}\, N(-d_1) + S_t e^{-q(T-t)} N'(d_1)\, \frac{\partial(-d_1)}{\partial S_t}$

(3) $= -X e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_2^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}} - e^{-q(T-t)}\, N(-d_1) - S_t e^{-q(T-t)} \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}}$

(4) $= e^{-q(T-t)}\, [N(d_1) - 1]$

Delta and gamma of a European call option and a European put option:

(5) $\frac{\partial^2 c_t}{\partial S_t^2} = \frac{\partial^2 p_t}{\partial S_t^2} = e^{-q(T-t)}\, N'(d_1)\, \frac{\partial d_1}{\partial S_t}$

(6) $= e^{-q(T-t)}\, \frac{1}{\sqrt{2\pi}} e^{\frac{-d_1^2}{2}}\, \frac{1}{S_t \sigma \sqrt{T-t}}$

Notice:

Equation (3) is a general result, which holds true, even if the trigger price levels X in d_1, and d_2, and the strike price X in the main equation, differ from each other.

Some basic calculus:

$\ln xy = \ln x + \ln y$ $\quad \ln \frac{x}{y} = \ln x - \ln y$ $\quad e^{f(x)} e^{g(x)} = e^{f(x)+g(x)}$

$D\Big[f(x)g(x)\Big] = f'(x)g(x) + f(x)g'(x)$

$D\Big[f[g(x)]\Big] = f'[g(x)]g'(x)$

$D\Big[f(x)^n\Big] = nf(x)^{n-1}f'(x)$ $\quad D\Big[e^{f(x)}\Big] = f'(x)e^{f(x)}$ $\quad D\Big[\ln f(x)\Big] = f'(x)\frac{1}{f(x)}$

	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
−2.4	0.0082	0.0081	0.0080	0.0079	0.0078	0.0077	0.0075	0.0074	0.0073	0.0072	0.0071	0.0070	0.0069	0.0069	0.0068	0.0067	0.0066	0.0065	0.0064	0.0063
−2.3	0.0107	0.0106	0.0104	0.0103	0.0102	0.0100	0.0099	0.0098	0.0096	0.0095	0.0094	0.0093	0.0091	0.0090	0.0089	0.0088	0.0087	0.0085	0.0084	0.0083
−2.2	0.0139	0.0137	0.0136	0.0134	0.0132	0.0130	0.0129	0.0127	0.0125	0.0124	0.0122	0.0121	0.0119	0.0118	0.0116	0.0115	0.0113	0.0112	0.0110	0.0109
−2.1	0.0179	0.0176	0.0174	0.0172	0.0170	0.0168	0.0166	0.0164	0.0162	0.0160	0.0158	0.0156	0.0154	0.0152	0.0150	0.0148	0.0146	0.0144	0.0143	0.0141
−2.0	0.0228	0.0225	0.0222	0.0220	0.0217	0.0214	0.0212	0.0209	0.0207	0.0204	0.0202	0.0199	0.0197	0.0195	0.0192	0.0190	0.0188	0.0185	0.0183	0.0181
−1.9	0.0287	0.0284	0.0281	0.0277	0.0274	0.0271	0.0268	0.0265	0.0262	0.0259	0.0256	0.0253	0.0250	0.0247	0.0244	0.0241	0.0239	0.0236	0.0233	0.0230
−1.8	0.0359	0.0355	0.0351	0.0348	0.0344	0.0340	0.0336	0.0333	0.0329	0.0325	0.0322	0.0318	0.0314	0.0311	0.0307	0.0304	0.0301	0.0297	0.0294	0.0290
−1.7	0.0446	0.0441	0.0436	0.0432	0.0427	0.0423	0.0418	0.0414	0.0409	0.0405	0.0401	0.0396	0.0392	0.0388	0.0384	0.0379	0.0375	0.0371	0.0367	0.0363
−1.6	0.0548	0.0542	0.0537	0.0532	0.0526	0.0521	0.0516	0.0510	0.0505	0.0500	0.0495	0.0490	0.0485	0.0480	0.0475	0.0470	0.0465	0.0460	0.0455	0.0450
−1.5	0.0668	0.0662	0.0655	0.0649	0.0643	0.0636	0.0630	0.0624	0.0618	0.0612	0.0606	0.0600	0.0594	0.0588	0.0582	0.0576	0.0571	0.0565	0.0559	0.0554
−1.4	0.0808	0.0800	0.0793	0.0785	0.0778	0.0771	0.0764	0.0756	0.0749	0.0742	0.0735	0.0728	0.0721	0.0715	0.0708	0.0701	0.0694	0.0688	0.0681	0.0675
−1.3	0.0968	0.0959	0.0951	0.0943	0.0934	0.0926	0.0918	0.0909	0.0901	0.0893	0.0885	0.0877	0.0869	0.0861	0.0853	0.0846	0.0838	0.0830	0.0823	0.0815
−1.2	0.1151	0.1141	0.1131	0.1122	0.1112	0.1103	0.1093	0.1084	0.1075	0.1066	0.1056	0.1047	0.1038	0.1029	0.1020	0.1012	0.1003	0.0994	0.0985	0.0977
−1.1	0.1357	0.1346	0.1335	0.1324	0.1314	0.1303	0.1292	0.1282	0.1271	0.1261	0.1251	0.1240	0.1230	0.1220	0.1210	0.1200	0.1190	0.1180	0.1170	0.1160
−1.0	0.1587	0.1574	0.1562	0.1551	0.1539	0.1527	0.1515	0.1503	0.1492	0.1480	0.1469	0.1457	0.1446	0.1434	0.1423	0.1412	0.1401	0.1390	0.1379	0.1368
−0.9	0.1841	0.1827	0.1814	0.1801	0.1788	0.1775	0.1762	0.1749	0.1736	0.1723	0.1711	0.1698	0.1685	0.1673	0.1660	0.1648	0.1635	0.1623	0.1611	0.1599
−0.8	0.2119	0.2104	0.2090	0.2075	0.2061	0.2047	0.2033	0.2019	0.2005	0.1991	0.1977	0.1963	0.1949	0.1935	0.1922	0.1908	0.1894	0.1881	0.1867	0.1854
−0.7	0.2420	0.2404	0.2389	0.2373	0.2358	0.2342	0.2327	0.2312	0.2296	0.2281	0.2266	0.2251	0.2236	0.2221	0.2206	0.2192	0.2177	0.2162	0.2148	0.2133
−0.6	0.2743	0.2726	0.2709	0.2693	0.2676	0.2660	0.2643	0.2627	0.2611	0.2595	0.2578	0.2562	0.2546	0.2530	0.2514	0.2498	0.2483	0.2467	0.2451	0.2435
−0.5	0.3085	0.3068	0.3050	0.3033	0.3015	0.2998	0.2981	0.2963	0.2946	0.2929	0.2912	0.2894	0.2877	0.2860	0.2843	0.2826	0.2810	0.2793	0.2776	0.2759
−0.4	0.3446	0.3427	0.3409	0.3391	0.3372	0.3354	0.3336	0.3318	0.3300	0.3282	0.3264	0.3246	0.3228	0.3210	0.3192	0.3174	0.3156	0.3138	0.3121	0.3103
−0.3	0.3821	0.3802	0.3783	0.3764	0.3745	0.3726	0.3707	0.3688	0.3669	0.3650	0.3632	0.3613	0.3594	0.3576	0.3557	0.3538	0.3520	0.3501	0.3483	0.3464
−0.2	0.4207	0.4188	0.4168	0.4149	0.4129	0.4110	0.4090	0.4071	0.4052	0.4032	0.4013	0.3994	0.3974	0.3955	0.3936	0.3917	0.3897	0.3878	0.3859	0.3840
−0.1	0.4602	0.4582	0.4562	0.4542	0.4522	0.4503	0.4483	0.4463	0.4443	0.4424	0.4404	0.4384	0.4364	0.4345	0.4325	0.4305	0.4286	0.4266	0.4247	0.4227
−0.0	0.5000	0.4980	0.4960	0.4940	0.4920	0.4900	0.4880	0.4860	0.4840	0.4821	0.4801	0.4781	0.4761	0.4741	0.4721	0.4701	0.4681	0.4661	0.4641	0.4622
0.0	0.5000	0.5020	0.5040	0.5060	0.5080	0.5100	0.5120	0.5140	0.5160	0.5179	0.5199	0.5219	0.5239	0.5259	0.5279	0.5299	0.5319	0.5339	0.5359	0.5378
0.1	0.5398	0.5418	0.5438	0.5458	0.5478	0.5497	0.5517	0.5537	0.5557	0.5576	0.5596	0.5616	0.5636	0.5655	0.5675	0.5695	0.5714	0.5734	0.5753	0.5773
0.2	0.5793	0.5812	0.5832	0.5851	0.5871	0.5890	0.5910	0.5929	0.5948	0.5968	0.5987	0.6006	0.6026	0.6045	0.6064	0.6083	0.6103	0.6122	0.6141	0.6160
0.3	0.6179	0.6198	0.6217	0.6236	0.6255	0.6274	0.6293	0.6312	0.6331	0.6350	0.6368	0.6387	0.6406	0.6424	0.6443	0.6462	0.6480	0.6499	0.6517	0.6536
0.4	0.6554	0.6573	0.6591	0.6609	0.6628	0.6646	0.6664	0.6682	0.6700	0.6718	0.6736	0.6754	0.6772	0.6790	0.6808	0.6826	0.6844	0.6862	0.6879	0.6897
0.5	0.6915	0.6932	0.6950	0.6967	0.6985	0.7002	0.7019	0.7037	0.7054	0.7071	0.7088	0.7106	0.7123	0.7140	0.7157	0.7174	0.7190	0.7207	0.7224	0.7241
0.6	0.7257	0.7274	0.7291	0.7307	0.7324	0.7340	0.7357	0.7373	0.7389	0.7405	0.7422	0.7438	0.7454	0.7470	0.7486	0.7502	0.7517	0.7533	0.7549	0.7565
0.7	0.7580	0.7596	0.7611	0.7627	0.7642	0.7658	0.7673	0.7688	0.7704	0.7719	0.7734	0.7749	0.7764	0.7779	0.7794	0.7808	0.7823	0.7838	0.7852	0.7867
0.8	0.7881	0.7896	0.7910	0.7925	0.7939	0.7953	0.7967	0.7981	0.7995	0.8009	0.8023	0.8037	0.8051	0.8065	0.8078	0.8092	0.8106	0.8119	0.8133	0.8146
0.9	0.8159	0.8173	0.8186	0.8199	0.8212	0.8225	0.8238	0.8251	0.8264	0.8277	0.8289	0.8302	0.8315	0.8327	0.8340	0.8352	0.8365	0.8377	0.8389	0.8401
1.0	0.8413	0.8426	0.8438	0.8449	0.8461	0.8473	0.8485	0.8497	0.8508	0.8520	0.8531	0.8543	0.8554	0.8566	0.8577	0.8588	0.8599	0.8610	0.8621	0.8632
1.1	0.8643	0.8654	0.8665	0.8676	0.8686	0.8697	0.8708	0.8718	0.8729	0.8739	0.8749	0.8760	0.8770	0.8780	0.8790	0.8800	0.8810	0.8820	0.8830	0.8840
1.2	0.8849	0.8859	0.8869	0.8878	0.8888	0.8897	0.8907	0.8916	0.8925	0.8934	0.8944	0.8953	0.8962	0.8971	0.8980	0.8988	0.8997	0.9006	0.9015	0.9023
1.3	0.9032	0.9041	0.9049	0.9057	0.9066	0.9074	0.9082	0.9091	0.9099	0.9107	0.9115	0.9123	0.9131	0.9139	0.9147	0.9154	0.9162	0.9170	0.9177	0.9185
1.4	0.9192	0.9200	0.9207	0.9215	0.9222	0.9229	0.9236	0.9244	0.9251	0.9258	0.9265	0.9272	0.9279	0.9285	0.9292	0.9299	0.9306	0.9312	0.9319	0.9325
1.5	0.9332	0.9338	0.9345	0.9351	0.9357	0.9364	0.9370	0.9376	0.9382	0.9388	0.9394	0.9400	0.9406	0.9412	0.9418	0.9424	0.9429	0.9435	0.9441	0.9446
1.6	0.9452	0.9458	0.9463	0.9468	0.9474	0.9479	0.9484	0.9490	0.9495	0.9500	0.9505	0.9510	0.9515	0.9520	0.9525	0.9530	0.9535	0.9540	0.9545	0.9550
1.7	0.9554	0.9559	0.9564	0.9568	0.9573	0.9577	0.9582	0.9586	0.9591	0.9595	0.9599	0.9604	0.9608	0.9612	0.9616	0.9621	0.9625	0.9629	0.9633	0.9637
1.8	0.9641	0.9645	0.9649	0.9652	0.9656	0.9660	0.9664	0.9667	0.9671	0.9675	0.9678	0.9682	0.9686	0.9689	0.9693	0.9696	0.9699	0.9703	0.9706	0.9710
1.9	0.9713	0.9716	0.9719	0.9723	0.9726	0.9729	0.9732	0.9735	0.9738	0.9741	0.9744	0.9747	0.9750	0.9753	0.9756	0.9759	0.9761	0.9764	0.9767	0.9770
2.0	0.9772	0.9775	0.9778	0.9780	0.9783	0.9786	0.9788	0.9791	0.9793	0.9796	0.9798	0.9801	0.9803	0.9805	0.9808	0.9810	0.9812	0.9815	0.9817	0.9819
2.1	0.9821	0.9824	0.9826	0.9828	0.9830	0.9832	0.9834	0.9836	0.9838	0.9840	0.9842	0.9844	0.9846	0.9848	0.9850	0.9852	0.9854	0.9856	0.9857	0.9859
2.2	0.9861	0.9863	0.9864	0.9866	0.9868	0.9870	0.9871	0.9873	0.9875	0.9876	0.9878	0.9879	0.9881	0.9882	0.9884	0.9885	0.9887	0.9888	0.9890	0.9891
2.3	0.9893	0.9894	0.9896	0.9897	0.9898	0.9900	0.9901	0.9902	0.9904	0.9905	0.9906	0.9907	0.9909	0.9910	0.9911	0.9912	0.9913	0.9915	0.9916	0.9917
2.4	0.9918	0.9919	0.9920	0.9921	0.9922	0.9923	0.9925	0.9926	0.9927	0.9928	0.9929	0.9930	0.9931	0.9931	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937