OULUN YLIOPISTO
UNIVERSITY of OULU

YLIOPISTOTENTTILOMAKEPOHJA / UNIVERSITY EXAM TEMPLATE

Koskee tiedekuntia LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit) /
Concerns Faculties SCI, OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

9.12.2015	Tentin kesto: 4 tuntia
Tiedekunta / Faculty:	**Oulun yliopiston kauppakorkeakoulu**
Opintojakson koodi, nimi ja tentin numero / The code and the name of the course and number of the exam: 721957S Fundamentals of Finance	
Tentaattori(t) / Examiner(s): **Jukka Perttunen**	**Sisäinen postios. / Internal address:** **6 OyKKK**

Sallitut apuvälineet / The devices allowed in the exam:

☒ Nelilaskin / Standard calculator
☒ Funktiolaskin / Scientific calculator
☒ Ohjelmoitava laskin / Programmable calculator
☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

☒ Suomeksi / in Finnish
☒ Englanniksi / in English

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

☐ Kyllä / Yes
☒ Ei / No

1. Probability density function $f(x) = 3x^2$ describes the relative likelihood of the random variable x within the range from $x = 0$ to $x = 1$.

   a) Show that the total probability between $x = 0$ and $x = 1$ equals to one.

   b) Determine the expected value of x.

   c) Determine the variance of x.

   d) Determine the probability $P(x \geq 0.5)$.

2. Asset price follows Geometric Brownian Motion. The expected asset price at the end of a six-month period is 15.86 euros, and the volatility of the asset price is 30%. The current asset price is 15 euros.

   a) Determine the standard deviation of the asset price at the end of the six-month period.

   c) Determine the probability that the asset price is at or above 15.86 euros at the end of the six-month period.

3. Three stocks together form a tangent portfolio corresponding to the risk-free rate of 2.0%. The weights of the three stocks in the tangent portfolio are $w_1 = 0.40$, $w_2 = 0.28$, and $w_3 = 0.32$, and the stocks trade currently at $S_1 = 12.50$, $S_2 = 7.95$, and $S_3 = 14.20$. The expected return of the tangent portfolio is 12.0%. Suppose that you create an efficient portfolio, worth of one million euros, with the expected return of 8.4% and the volatility of 16.0%.

   a) Determine the amount of cash invested in the risk-free asset.

   b) Determine the volatility of the tangent portfolio.

   c) Determine the number of shares of asset #1 in the portfolio.

4. The return variances and the covariance of a stock i and the market portfolio m are estimated over a three-year period of daily data (750 observations) and are $s_i^2 = 0.0005041$, $s_m^2 = 0.0001369$, and $s_{im} = 0.0001766$.

   a) Determine the (annual) volatility of the stock i.

   b) Determine the beta of the stock i.

   c) Determine the (annual) volatility of the equally weighted portfolio of the stock and the market portfolio.

5. The unlevered free cash from the previous year is 200 thousand euros. The cash flow is expected to grow at a 10% annual rate over the next two years. From the beginning of the third year the growth rate is expected to stabilize at the level of 5%. The required rate of return on assets is 12% and the debt-to-equity ratio of the firm is 1.5. Determine the market value of equity of the firm.

$$y = f(x) = ax^n \qquad y' = f'(x) = \frac{dy}{dx} = anx^{n-1}$$

$$y = f(x) = ae^x \qquad y' = f'(x) = \frac{dy}{dx} = ae^x$$

$$y = f(x) = a\ln x \qquad y' = f'(x) = \frac{dy}{dx} = \frac{a}{x}$$

$$y = f(x)g(x) \qquad y' = f'(x)g(x) + f(x)g'(x)$$

$$y = f[g(x)] \qquad y' = f'[g(x)]g'(x)$$

$$y = af(x)^n \qquad y' = anf(x)^{n-1}f'(x)$$

$$y = ae^{f(x)} \qquad y' = ae^{f(x)}f'(x)$$

$$y = a\ln f(x) \qquad y' = \frac{a}{f(x)}f'(x)$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)dx$$

$$Var(x) = \int_{-\infty}^{\infty} [x - E(x)]^2 f(x)dx$$

$$P(x \le a) = \int_{-\infty}^{a} f(x)dx$$

$$E(S_T) = S_0 e^{\mu T}$$

$$Std(S_T) = S_0 e^{\mu T}\sqrt{e^{\sigma^2 T} - 1}$$

$$E(\ln S_T) = \ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T$$

$$Std(\ln S_T) = \sigma\sqrt{T}$$

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

$$Var(R_p) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}$$

$$\sum_{t=1}^{\infty} \frac{(1+g)^t D_0}{(1+k)^t} = \frac{(1+g)D_0}{k-g} = \frac{D_1}{k-g}$$

	00	05	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
−2.4	0.0082	0.0081	0.0080	0.0079	0.0078	0.0077	0.0075	0.0074	0.0073	0.0072	0.0071	0.0070	0.0069	0.0069	0.0068	0.0067	0.0066	0.0065	0.0064	0.0063
−2.3	0.0107	0.0106	0.0104	0.0103	0.0102	0.0100	0.0099	0.0098	0.0096	0.0095	0.0094	0.0093	0.0091	0.0090	0.0089	0.0088	0.0087	0.0085	0.0084	0.0083
−2.2	0.0139	0.0137	0.0136	0.0134	0.0132	0.0130	0.0129	0.0127	0.0125	0.0124	0.0122	0.0121	0.0119	0.0118	0.0116	0.0115	0.0113	0.0112	0.0110	0.0109
−2.1	0.0179	0.0176	0.0174	0.0172	0.0170	0.0168	0.0166	0.0164	0.0162	0.0160	0.0158	0.0156	0.0154	0.0152	0.0150	0.0148	0.0146	0.0144	0.0143	0.0141
−2.0	0.0228	0.0225	0.0222	0.0220	0.0217	0.0214	0.0212	0.0209	0.0207	0.0204	0.0202	0.0199	0.0197	0.0195	0.0192	0.0190	0.0188	0.0185	0.0183	0.0181
−1.9	0.0287	0.0284	0.0281	0.0277	0.0274	0.0271	0.0268	0.0265	0.0262	0.0259	0.0256	0.0253	0.0250	0.0247	0.0244	0.0241	0.0239	0.0236	0.0233	0.0230
−1.8	0.0359	0.0355	0.0351	0.0348	0.0344	0.0340	0.0336	0.0333	0.0329	0.0325	0.0322	0.0318	0.0314	0.0311	0.0307	0.0304	0.0301	0.0297	0.0294	0.0290
−1.7	0.0446	0.0441	0.0436	0.0432	0.0427	0.0423	0.0418	0.0414	0.0409	0.0405	0.0401	0.0396	0.0392	0.0388	0.0384	0.0379	0.0375	0.0371	0.0367	0.0363
−1.6	0.0548	0.0542	0.0537	0.0532	0.0526	0.0521	0.0516	0.0510	0.0505	0.0500	0.0495	0.0490	0.0485	0.0480	0.0475	0.0470	0.0465	0.0460	0.0455	0.0450
−1.5	0.0668	0.0662	0.0655	0.0649	0.0643	0.0636	0.0630	0.0624	0.0618	0.0612	0.0606	0.0600	0.0594	0.0588	0.0582	0.0576	0.0571	0.0565	0.0559	0.0554
−1.4	0.0808	0.0800	0.0793	0.0785	0.0778	0.0771	0.0764	0.0756	0.0749	0.0742	0.0735	0.0728	0.0721	0.0715	0.0708	0.0701	0.0694	0.0688	0.0681	0.0675
−1.3	0.0968	0.0959	0.0951	0.0943	0.0934	0.0926	0.0918	0.0909	0.0901	0.0893	0.0885	0.0877	0.0869	0.0861	0.0853	0.0846	0.0838	0.0830	0.0823	0.0815
−1.2	0.1151	0.1141	0.1131	0.1122	0.1112	0.1103	0.1093	0.1084	0.1075	0.1066	0.1056	0.1047	0.1038	0.1029	0.1020	0.1012	0.1003	0.0994	0.0985	0.0977
−1.1	0.1357	0.1346	0.1335	0.1324	0.1314	0.1303	0.1292	0.1282	0.1271	0.1261	0.1251	0.1240	0.1230	0.1220	0.1210	0.1200	0.1190	0.1180	0.1170	0.1160
−1.0	0.1587	0.1574	0.1562	0.1551	0.1539	0.1527	0.1515	0.1503	0.1492	0.1480	0.1469	0.1457	0.1446	0.1434	0.1423	0.1412	0.1401	0.1390	0.1379	0.1368
−0.9	0.1841	0.1827	0.1814	0.1801	0.1788	0.1775	0.1762	0.1749	0.1736	0.1723	0.1711	0.1698	0.1685	0.1673	0.1660	0.1648	0.1635	0.1623	0.1611	0.1599
−0.8	0.2119	0.2104	0.2090	0.2075	0.2061	0.2047	0.2033	0.2019	0.2005	0.1991	0.1977	0.1963	0.1949	0.1935	0.1922	0.1908	0.1894	0.1881	0.1867	0.1854
−0.7	0.2420	0.2404	0.2389	0.2373	0.2358	0.2342	0.2327	0.2312	0.2296	0.2281	0.2266	0.2251	0.2236	0.2221	0.2206	0.2192	0.2177	0.2162	0.2148	0.2133
−0.6	0.2743	0.2726	0.2709	0.2693	0.2676	0.2660	0.2643	0.2627	0.2611	0.2595	0.2578	0.2562	0.2546	0.2530	0.2514	0.2498	0.2483	0.2467	0.2451	0.2435
−0.5	0.3085	0.3068	0.3050	0.3033	0.3015	0.2998	0.2981	0.2963	0.2946	0.2929	0.2912	0.2894	0.2877	0.2860	0.2843	0.2826	0.2810	0.2793	0.2776	0.2759
−0.4	0.3446	0.3427	0.3409	0.3391	0.3372	0.3354	0.3336	0.3318	0.3300	0.3282	0.3264	0.3246	0.3228	0.3210	0.3192	0.3174	0.3156	0.3138	0.3121	0.3103
−0.3	0.3821	0.3802	0.3783	0.3764	0.3745	0.3726	0.3707	0.3688	0.3669	0.3650	0.3632	0.3613	0.3594	0.3576	0.3557	0.3538	0.3520	0.3501	0.3483	0.3464
−0.2	0.4207	0.4188	0.4168	0.4149	0.4129	0.4110	0.4090	0.4071	0.4052	0.4032	0.4013	0.3994	0.3974	0.3955	0.3936	0.3917	0.3897	0.3878	0.3859	0.3840
−0.1	0.4602	0.4582	0.4562	0.4542	0.4522	0.4503	0.4483	0.4463	0.4443	0.4424	0.4404	0.4384	0.4364	0.4345	0.4325	0.4305	0.4286	0.4266	0.4247	0.4227
−0.0	0.5000	0.4980	0.4960	0.4940	0.4920	0.4900	0.4880	0.4860	0.4840	0.4821	0.4801	0.4781	0.4761	0.4741	0.4721	0.4701	0.4681	0.4661	0.4641	0.4622
0.0	0.5000	0.5020	0.5040	0.5060	0.5080	0.5100	0.5120	0.5140	0.5160	0.5179	0.5199	0.5219	0.5239	0.5259	0.5279	0.5299	0.5319	0.5339	0.5359	0.5378
0.1	0.5398	0.5418	0.5438	0.5458	0.5478	0.5497	0.5517	0.5537	0.5557	0.5576	0.5596	0.5616	0.5636	0.5655	0.5675	0.5695	0.5714	0.5734	0.5753	0.5773
0.2	0.5793	0.5812	0.5832	0.5851	0.5871	0.5890	0.5910	0.5929	0.5948	0.5968	0.5987	0.6006	0.6026	0.6045	0.6064	0.6083	0.6103	0.6122	0.6141	0.6160
0.3	0.6179	0.6198	0.6217	0.6236	0.6255	0.6274	0.6293	0.6312	0.6331	0.6350	0.6368	0.6387	0.6406	0.6424	0.6443	0.6462	0.6480	0.6499	0.6517	0.6536
0.4	0.6554	0.6573	0.6591	0.6609	0.6628	0.6646	0.6664	0.6682	0.6700	0.6718	0.6736	0.6754	0.6772	0.6790	0.6808	0.6826	0.6844	0.6862	0.6879	0.6897
0.5	0.6915	0.6932	0.6950	0.6967	0.6985	0.7002	0.7019	0.7037	0.7054	0.7071	0.7088	0.7106	0.7123	0.7140	0.7157	0.7174	0.7190	0.7207	0.7224	0.7241
0.6	0.7257	0.7274	0.7291	0.7307	0.7324	0.7340	0.7357	0.7373	0.7389	0.7405	0.7422	0.7438	0.7454	0.7470	0.7486	0.7502	0.7517	0.7533	0.7549	0.7565
0.7	0.7580	0.7596	0.7611	0.7627	0.7642	0.7658	0.7673	0.7688	0.7704	0.7719	0.7734	0.7749	0.7764	0.7779	0.7794	0.7808	0.7823	0.7838	0.7852	0.7867
0.8	0.7881	0.7896	0.7910	0.7925	0.7939	0.7953	0.7967	0.7981	0.7995	0.8009	0.8023	0.8037	0.8051	0.8065	0.8078	0.8092	0.8106	0.8119	0.8133	0.8146
0.9	0.8159	0.8173	0.8186	0.8199	0.8212	0.8225	0.8238	0.8251	0.8264	0.8277	0.8289	0.8302	0.8315	0.8327	0.8340	0.8352	0.8365	0.8377	0.8389	0.8401
1.0	0.8413	0.8426	0.8438	0.8449	0.8461	0.8473	0.8485	0.8497	0.8508	0.8520	0.8531	0.8543	0.8554	0.8566	0.8577	0.8588	0.8599	0.8610	0.8621	0.8632
1.1	0.8643	0.8654	0.8665	0.8676	0.8686	0.8697	0.8708	0.8718	0.8729	0.8739	0.8749	0.8760	0.8770	0.8780	0.8790	0.8800	0.8810	0.8820	0.8830	0.8840
1.2	0.8849	0.8859	0.8869	0.8878	0.8888	0.8897	0.8907	0.8916	0.8925	0.8934	0.8944	0.8953	0.8962	0.8971	0.8980	0.8988	0.8997	0.9006	0.9015	0.9023
1.3	0.9032	0.9041	0.9049	0.9057	0.9066	0.9074	0.9082	0.9091	0.9099	0.9107	0.9115	0.9123	0.9131	0.9139	0.9147	0.9154	0.9162	0.9170	0.9177	0.9185
1.4	0.9192	0.9200	0.9207	0.9215	0.9222	0.9229	0.9236	0.9244	0.9251	0.9258	0.9265	0.9272	0.9279	0.9285	0.9292	0.9299	0.9306	0.9312	0.9319	0.9325
1.5	0.9332	0.9338	0.9345	0.9351	0.9357	0.9364	0.9370	0.9376	0.9382	0.9388	0.9394	0.9400	0.9406	0.9412	0.9418	0.9424	0.9429	0.9435	0.9441	0.9446
1.6	0.9452	0.9458	0.9463	0.9468	0.9474	0.9479	0.9484	0.9490	0.9495	0.9500	0.9505	0.9510	0.9515	0.9520	0.9525	0.9530	0.9535	0.9540	0.9545	0.9550
1.7	0.9554	0.9559	0.9564	0.9568	0.9573	0.9577	0.9582	0.9586	0.9591	0.9595	0.9599	0.9604	0.9608	0.9612	0.9616	0.9621	0.9625	0.9629	0.9633	0.9637
1.8	0.9641	0.9645	0.9649	0.9652	0.9656	0.9660	0.9664	0.9667	0.9671	0.9675	0.9678	0.9682	0.9686	0.9689	0.9693	0.9696	0.9699	0.9703	0.9706	0.9710
1.9	0.9713	0.9716	0.9719	0.9723	0.9726	0.9729	0.9732	0.9735	0.9738	0.9741	0.9744	0.9747	0.9750	0.9753	0.9756	0.9759	0.9761	0.9764	0.9767	0.9770
2.0	0.9772	0.9775	0.9778	0.9780	0.9783	0.9786	0.9788	0.9791	0.9793	0.9796	0.9798	0.9801	0.9803	0.9805	0.9808	0.9810	0.9812	0.9815	0.9817	0.9819
2.1	0.9821	0.9824	0.9826	0.9828	0.9830	0.9832	0.9834	0.9836	0.9838	0.9840	0.9842	0.9844	0.9846	0.9848	0.9850	0.9852	0.9854	0.9856	0.9857	0.9859
2.2	0.9861	0.9863	0.9864	0.9866	0.9868	0.9870	0.9871	0.9873	0.9875	0.9876	0.9878	0.9879	0.9881	0.9882	0.9884	0.9885	0.9887	0.9888	0.9890	0.9891
2.3	0.9893	0.9894	0.9896	0.9897	0.9898	0.9900	0.9901	0.9902	0.9904	0.9905	0.9906	0.9907	0.9909	0.9910	0.9911	0.9912	0.9913	0.9915	0.9916	0.9917
2.4	0.9918	0.9919	0.9920	0.9921	0.9922	0.9923	0.9925	0.9926	0.9927	0.9928	0.9929	0.9930	0.9931	0.9931	0.9932	0.9933	0.9934	0.9935	0.9936	0.9937