UNIVERSITY OF OULU

YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

Opintojakson koodi and nimi / The code and the name of the course:

Koodi / Code 721342S

Tentin nimi / Exam name: Game Theory

Tiedekunta / Faculty: OBS

Tentin pvm / Date of exam: 8.9.2016	**Tentin kesto tunteina / Exam in hours: 3**
Tentin nro / No. of the exam: Tentti	**Opintopistemäärä / Credit units: 6**
Tentaattori(t) / Examiner(s): Marja-Liisa Halko	**Sisäinen postios. / Internal address:** 6 OyKKK

Sallitut apuvälineet / The devices allowed in the exam:

☒ Nelilaskin / Standard calculator ☐ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:
☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Kysymyspaperi on palautettava / Paper with exam questions must be returned:
☒ Kyllä / Yes ☐ Ei / No

Game Theory (721342S), University of Oulu

Exam 8.9.2016

Answer all the questions (1-5). NOTE! As you will answer the question 5 in this question paper, please write below also your name and your student identification number and return this paper with your answers.

Name: ____

Student id. number: ____

1. Consider the normal form game below. Solve all Nash equilibria of the game (both pure strategy and mixed strategy equilibria). (6 points)

Player 2

Player 1		A	B
	A	1,4	2,0
	B	0,8	3,9

2. Consider the two-player game below:

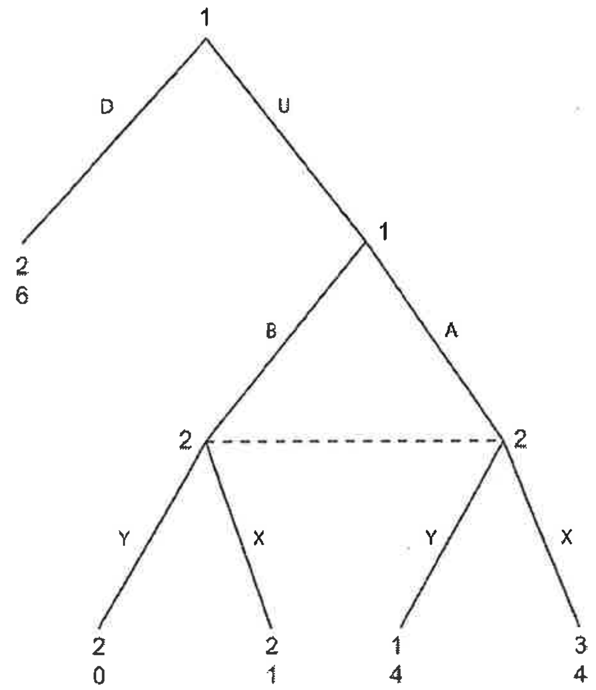
Player 2

Player 1		L	C	R
	U	5,1	1,4	1,0
	M	3,2	0,0	3,5
	D	4,3	4,4	0,3

(a) Show that Player 2's mixed strategy $\sigma_2 = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ strongly dominates Player 2's strategy L. (3 points)

(b) In addition, show that Player 2 has also other mixes strategies that strongly dominate the strategy L, and that actually, there exists an infinite number of such mixed strategies. (3 points)

3. Consider the two-player game below.



(a) Write the game in normal form and solve its Nash-equilibria.

(b) Solve the subgame perfect equilibria of the game. Which of the Nash-equilibria are nor subgame perfect?

(6 points)

4. Two people are working in a joint project, which will be finished, if and only if both work hard. Both prefer a situation in which both work hard, to a situation in which both goof off. Both, however, find the worst a situation in which they themselves work hard, but the other person goofs off. The following game describes the preferences of the players:

		Person 2	
		Goof off	Work hard
Person1	Goof off	0,0	0,-c
	Work hard	-c,0	1-c,1-c

In the game matrix, c is a positive number, smaller than one, illustrating the costs of working hard.

(a) Solve all mixed-strategy equilibria of the game. (4 points)

(b) How equilibria change, when c changes? (2 points)

5. Consider the following battle-of-sexes game. The husband (M) and the wife (V) have to decide whether they go to a football match (J) or to the opera (O). The husband prefers the football match and the wife opera. Assume that the husband is not sure about the preferences of the wife. In particular, he does not know whether she wants to be with the husband (loving) of prefers to go to the events alone (not-loving). In other words, he does not know whether the preferences of the wife are described by the game (a) or by the game (b). The husband believes that with probability ρ her preferences are described by the game (a) and with probability $1-\rho$ by the game (b). The wife knows her preferences and the beliefs of the husband.

(a) loving wife

		Wife J	Wife O
Husband	J	3,1	0,0
	O	0,0	1,3

(a) not-loving wife

		Wife J	Wife O
Husband	J	3,0	0,1
	O	0,3	1,0

On the next page, there are six statements relating to the solving the exercise. Indicate whether these statements are correct or incorrect. **Every right answer will give you one (1) point, and every wrong answer will give you one minus point (-1). You will get zero points if you do not answer anything.**

When we draw a game tree describing the decision making situation, the tree begins with a choice node of the "nature". From the first node, we draw two branches, one of which leads to the game (a) and the other to the game (b). Both games start from the husband's choice node, and two branches, J and O, start from each node.

STATEMENT 1: The choice nodes of the husband belong to the same information set and are combined with a dashed line, because the husband does not know which of the two games is played.

☐ Correct ☐ Incorrect

The branches starting from the choice nodes of the husband lead to the choice nodes of the wife. Thus, the wife has four choice nodes. Two branches, J and O, start from each four nodes. Finally, the payoffs corresponding the choices of the players are added to the game tree from game matrices (a) and (b).

STATEMENT 2: All four choice nodes of the wife are combined with a dashed line, because the wife does not know what the husband chose.

☐ Correct ☐ Incorrect

Next we write the game is a Bayesian normal form. We first form the strategies of the players.

STATEMENT 3: Both the husband and the wife have four strategies.

☐ Correct ☐ Incorrect

We add the Bayesian normal form the payoffs of the players.

STATEMENT 4: Payoffs depend on the probability ρ.

☐ Correct ☐ Incorrect

From the Bayesian normal form we solve the Nash-equilibria.

STATEMENT 5: In equilibrium, neither player deviates and chooses something other than the equilibrium strategy. Probability ρ does not affect the equilibria of the game.

☐ Correct ☐ Incorrect

We slightly change the decision making situation. Let us assume that the Wife does not know her own type, that is, also the Wife does not know whether game (a) or (b) is played.

STATEMENT 6: The change will affect the number of the strategies of the wife but not the number of the husband's strategies. In the new situation, wife has two strategies.

☐ Correct ☐ Incorrect