LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit)
SCI, OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

UNIVERSITY OF OULU

YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

Opintojakson koodi and nimi / The code and the name of the course:

Koodi / Code 721342S

Tentin nimi / Exam name Game Theory

Tiedekunta / Faculty:

Tentin pvm / Date of exam: 17.10.2016	**Tentin kesto tunteina / Exam in hours: 3**
Tentin nro / No. of the exam: 2. retake (esim. Tentti, 1. uusinta, 2. uusinta / e.g. Exam, 1. retake, 2. retake)	**Opintopistemäärä / Credit units: 6**
Tentaattori(t) / Examiner(s): Marja-Liisa Halko **Politiikan ja talouden tutkimuksen laitos**	**Sisäinen postios. / Internal address:** **PL 17, 00014 Helsingin yliopisto**

Sallitut apuvälineet / The devices allowed in the exam:

☒ Nelilaskin / Standard calculator
☒ Funktiolaskin / Scientific calculator
☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:
☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintojakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:
☒ Kyllä / Yes ☐ Ei / No

Answer all the questions (1-5). NOTE! As you will answer the question 5 in this question paper, please return this paper with your answers.

1. Player 1 is a police officer who must decide whether to patrol the streets or to hang out at the coffee shop. His payoff from hanging out at the coffee shop is 10, while his payoff from patrolling the streets depends on whether he catches a robber, who is player 2. If the robber prowls the streets then the police officer will catch him and obtain a payoff of 20. If the robber stays in his hideaway then the officer's payoff is 0. The robber must choose between staying hidden or prowling the streets. If he stays hidden then his payoff is 0, while if he prowls the streets his payoff is -10 if the officer is patrolling the streets and 10 if the officer is at the coffee shop.

(a) Write down the matrix form of this game. (2 points)

(b) Solve all Nash equilibria of the game (both pure strategy and mixed strategy equilibria). (4 points)

2. Consider the two-player game below:

Player 2

		L	C	R
Player 1	U	5,1	1,4	1,0
	M	3,2	0,0	3,5
	D	4,3	4,4	0,3

(a) Show that Player 2's mixed strategy $\sigma_2 = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ strongly dominates Player 2's strategy L. (3 points)

(b) In addition, show that Player 2 has also other mixes strategies that strongly dominate the strategy L, and that actually, there exists an infinite number of such mixed strategies. (3 points)

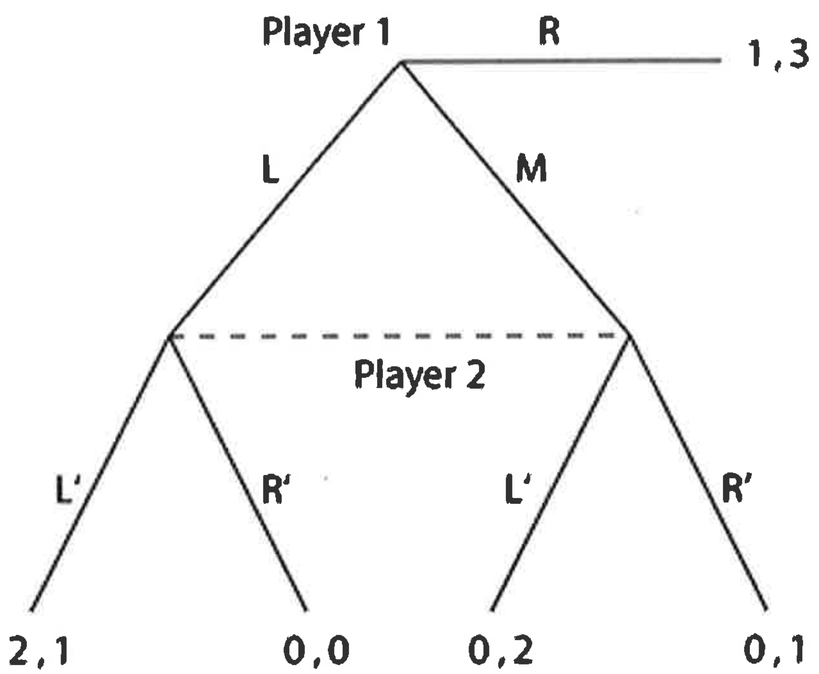
3. Two people select a policy that affects them both by alternately vetoing policies until one remains. First person 1 vetoes a policy. If more than one policy remains, person 2 then vetoes a policy. If more than one policy still remains, person 1 then vetoes a policy. The process continues until a single policy remains unvetoed. Suppose there are three possible policies, X, Y, and Z, person 1 prefers X to Y to Z, and person 2 prefers Z to Y to X.

(a) Model this situation as an extensive game. (2 points)

(b) Write the game in normal form and solve its Nash equilibria. (2 points)

(c) Solve the subgame perfect equilibria of the game. Does the game have Nash equilibria that are not subgame perfect equilibria? (2 points)

4. Write the game below in normal form and solve its Nash equilibria (in pure strategies). (6 points)



5. Two persons are involved in a dispute. Person 1 does not know whether person 2 is weak or strong; she assigns probability α to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Both will get a payoff of 0 if they yield (regardless of the other person's action) and a payoff of 1 if she fights and her opponent yields. If both players fight, then their payoffs are (-1, 1), if person 2 is strong, and (1,-1) if person 2 is weak.

(a) Formulate the decision-making situation as a Bayesian game, that is, write the game matrices that describe the situation. (4 points)

(b) In the following, there are six statements relating to the solution of the problem. Indicate whether these statements are true of false. **For every correct answer you will get two (2) points and for every wrong answer you will get minus one point (-1). If you do not answer anything, you will get zero points. Note! However, the lower limit of your points is zero points.** (max 12 points)

Write first the Bayesian normal form of the game. Let's first write the strategies of the players.

STATEMENT 1: Person 1 has two strategies, because she does not know, whether person 2 is weak of strong.

☐ True ☐ False

STATEMENT 2: Person 2 has two strategies, because she knows whether she herself is weak or strong.

☐ True ☐ False

Next we add the Bayesian normal form the payoffs of the players.

STATEMENT 3: Both person 1's and person 2's payoffs depend on the probability α.

☐ True ☐ False

STATEMENT 4: If both players always fight, person 1 gets payoff $1 - 2\alpha$ and person 2 gets payoff $2\alpha - 1$.

☐ True ☐ False

Next we use the Bayesian normal form to solve the Nash equilibria of the game (or bayesian Nash equilibria). In the equilibrium, neither of the players wants to deviate from the equilibrium strategy and choose some other strategy.

STATEMENT 5: If $> \frac{1}{2}$, then, in the equilibrium, person 1 always yields and person 2 always fights.

☐ True ☐ False

STATEMENT 6: If $\alpha < \frac{1}{2}$, the game does not have any Nash equilibria.

☐ True ☐ False