YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

Opintojakson koodi and nimi / The code and the name of the course:	
Koodi / Code 721342S **Tentin nimi / Exam name Game Theory**	
Tiedekunta / Faculty:	
Tentin pvm / Date of exam: 29.5.2017	**Tentin kesto tunteina / Exam in hours: 3**
Tentin nro / No. of the exam: 1. uusinta (esim. Tentti, 1. uusinta, 2. uusinta / e.g. Exam, 1. retake, 2. retake)	**Opintopistemäärä / Credit units: 6**
Tentaattori(t) / Examiner(s): Marja-Liisa Halko **Politiikan ja talouden tutkimuksen laitos**	**Sisäinen postios. / Internal address:** **PL 17, 00014 Helsingin yliopisto**

Sallitut apuvälineet / The devices allowed in the exam:

☒ Nelilaskin / Standard calculator ☒ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintojakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

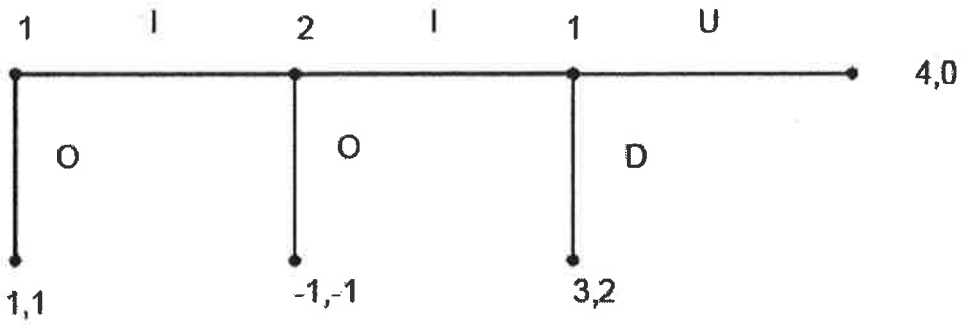
☒ Kyllä / Yes ☐ Ei / No

Answer all the questions (1-5). NOTE! As you will answer the question 5 in this question paper, please return this paper with your answers.

1. Consider a lobbying game between two firms. Each firm may lobby the government in hopes of persuading the government to make a decision that is favorable to the firm. The two firms, X and Y, independently and simultaneously decide whether to lobby (L) or not (N). Lobbying entails a cost of 15. Not lobbying costs nothing. If both firms lobby of neither firm lobbies then the government takes a neutral decision, which yields 10 to both firms. (A firm's payoff is this value minus the lobbying cost, if it lobbied.) If firm Y lobbies and firm X does not lobby, then the government makes a decision that favors firm Y, yielding zero to firm X and 30 to firm Y. If firm X lobbies and firm Y does not lobby, then the government makes a decision that favors firm X, yielding 40 to firm X and zero to firm Y.

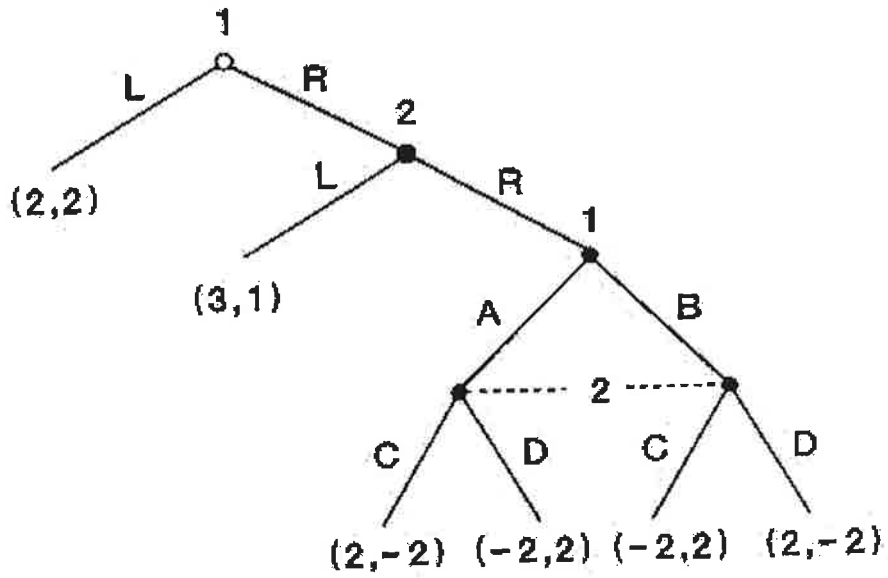
(a) Write the normal form of this game. (2 points)
(b) Next solve its pure strategy Nash equilibria. (3 points)

2. Consider the following extensive form game:



(a) Write the game in normal form and solve its Nash equilibria. (3 points)
(b) Solve the subgame perfect equilibria of the game. Does the game have Nash equilibria that are not subgame perfect equilibria? (3 points)

3. Write the game below in normal form and solve its Nash equilibria (in pure strategies). (5 points)



4. Consider the two-player game below:

		Player 2		
		L	C	R
Player 1	U	5,1	1,4	1,0
	M	3,2	0,0	3,5
	D	4,3	4,4	0,3

(a) Does either of the players have dominant strategies in pure strategies? Explain. (3 points)

(b) Next show that Player 2's mixed strategy $\sigma_2 = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ strongly dominates Player 2's strategy L. (3 points)

(b) In addition, show that Player 2 has also other mixes strategies that strongly dominate the strategy L, and that actually, there exists an infinite number of such mixed strategies. (3 points)

5. Consider the following Battle of sexes –game between a husband and a wife:

(a) Loving

		Wife F	Wife O
Husband	F	3,1	0,0
	O	0,0	1,3

(b) Leaving

		Wife F	Wife O
Husband	F	3,0	0,1
	O	0,3	1,0

The husband prefers F to O, but would rather be together than apart. The husband is not sure of his wife's preferences. He does not know if his wife likes to be with him (loving) or if she prefers to go either event by herself (leaving). The wife knows her own preferences. The husband believes that with probability ρ the wife is loving and with probability $1 - \rho$ she is leaving. The wife knows the beliefs of the husband (knows ρ, common prior).

a) First draw the game in an extensive form, where an artificial player, "nature", first selects the type of the wife. Remember to be careful with the information sets. (4 points)
b) Does any player (or any player type) have a dominant strategy? Explain. (4 points)

(c) In the following, there are six statements relating to the solution of the problem. Indicate whether these statements are true of false. **For every correct answer you will get two (2) points and for every wrong answer you will get minus one point (-1). If you do not answer anything, you will get zero points. Note! However, the lower limit of your points is zero points.** (max 12 points)

Write next the Bayesian normal form of the game. Let's first write the strategies of the players.

STATEMENT 1: Husband has four strategies, because he does not know, whether his wife is loving or leaving.

☐ True ☐ False

STATEMENT 2: Wife has four strategies, because she knows whether she is loving or leaving.

☐ True ☐ False

STATEMENT 3: Husband's payoffs depend on the probability ρ because he does not know, whether his wife is loving or leaving, but the wife's payoffs do not depend on the probability ρ.

☐ True ☐ False

STATEMENT 4: If both players always choose F, the payoff of the husband is 3 and the payoff of the wife is ρ.

☐ True ☐ False

Next we use the Bayesian normal form to solve the Nash equilibria of the game (or bayesian Nash equilibria). In the equilibrium, neither of the players wants to deviate from the equilibrium strategy and choose some other strategy.

STATEMENT 5: If $\rho < \frac{1}{4}$, then the game does not have a Nash equilibrium.

☐ True ☐ False

STATEMENT 6: If $\rho \geq \frac{1}{4}$, there exists an equilibrium where both husband and wife always choose F.

☐ True ☐ False