



## YLIOPISTOTENTTILOMAKEPOHJA / UNIVERSITY EXAM TEMPLATE

Koskee tiedekuntia LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit) /  
Concerns Faculties SCI, OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

<b>Tentin päivämäärä / Date of exam:</b> 30.11.2015	<b>Tentin kesto tunteina / Exam in hours:</b> 4
<b>Tiedekunta / Faculty:</b> Oulu Business School	
<b>Opintojakson koodi, nimi ja tentin numero / The code and the name of the course and number of the exam:</b> 721338S, Mathematical Economics, 6 credits (2 <sup>nd</sup> exam)	
<b>Tentaattori(t) / Examiner(s):</b> Matti Koivuranta	<b>Sisäinen postios. / Internal address</b> 6 Oulu Business School
<b>Sallitut apuvälineet / The devices allowed in the exam:</b>	
<input checked="" type="checkbox"/> Nelilaskin / Standard calculator	<input type="checkbox"/> Funktiolaskin / Scientific calculator
<input type="checkbox"/> Ohjelmoitava laskin / Programmable calculator	
<input type="checkbox"/> Muu materiaali, tarkennettu alla / Other material, specified below:	
<b>Tenttiin vastaaminen / Please answer the questions:</b>	
<input checked="" type="checkbox"/> Suomeksi / in Finnish	<input checked="" type="checkbox"/> Englanniksi / in English
<b>Kysymyspaperi on palautettava / Paper with exam questions must be returned:</b>	
<input type="checkbox"/> Kyllä / Yes	<input checked="" type="checkbox"/> Ei / No

*All five questions should be answered*

1. Assume following matrices:

$$A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -4 \\ 2 & -2 \end{bmatrix}.$$

Calculate the following:

- sum  $A + B$
  - transpose  $B^T$
  - product  $AB$
  - determinant  $|A|$
  - inverse  $A^{-1}$
  - rank of matrix  $B$
2. a) Solve equation  $\frac{x}{4} = \sqrt{x}$ .  
b) Differentiate function  $f(x) = \ln\left(\frac{1}{x}\right)$ .  
c) Compute definite integral  $\int_{-1}^1 (x^2 - 1)dx$ .
3. Equation  $e^{y+x} = y$  holds when  $x = 1 - e$  and  $y = e$ . Does the equation determine  $y$  as a function of  $x$  around point  $x = 1 - e$ ? If yes, calculate the derivative  $\frac{dy}{dx}$  at point  $x = 1 - e$ .
4. Solve the utility maximization problem

$$\max_{\{x_1, x_2\}} \{x_1^\alpha x_2^\beta\} \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq w$$

for the optimal consumption bundle  $(x_1^*, x_2^*) \in \mathbb{R}_+^2$ . Parameters  $\alpha, \beta, p_1, p_2$  and  $w$  are all strictly positive.

5. a) Find an explicit solution for difference equation  $y_{t+1} = y_t + 1$  with initial condition  $y_0 = 0$ .  
b) Find stationary points of differential equation  $y'(t) = y(t)^3 - y(t)$  and use graphical analysis to determine whether the stationary points are locally stable or not. Sketch the solution function  $y(t)$  assuming initial condition  $y(0) = \frac{1}{2}$ .

Following lemmas, which consider testing for local quality of critical points, may or may not be useful:

**Lemma 1 (definiteness of Hessian)** *Matrix  $A$  is*

1. *positive definite, if  $|A_i| > 0 \quad \forall i = 1, \dots, n$ .*
2. *negative definite, if  $|A_1| < 0, |A_2| > 0, |A_3| < 0$ , etc. (sign alternates with first sign negative).*
3. *positive semi-definite, if  $|A_i| \geq 0 \quad \forall i = 1, \dots, n$ .*
4. *negative semi-definite, if  $|A_1| \leq 0, |A_2| \geq 0, |A_3| \leq 0$ , etc.*
5. *indefinite, if none of the above holds.*

**Lemma 2 (using bordered Hessian)** *Assume an equality-constrained optimization problem with  $n$  choice variables and  $m$  constraints. Critical points of the Lagrangian can be classified by examining subdeterminants of the bordered Hessian.*

*A critical point is a strict local minimum if the signs of the  $n - m$  largest subdeterminants are all equal to the sign of  $(-1)^m$ . A critical point is a strict local maximum if the signs of  $n - m$  largest subdeterminants alternate and the sign of the largest subdeterminant is equal to the sign of  $(-1)^n$ .*

