OULUN YLIOPISTO
UNIVERSITY of OULU

YLIOPISTOTENTTILOMAKEPOHJA / UNIVERSITY EXAM TEMPLATE

Koskee tiedekuntia LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit) /
Concerns Faculties SCI, OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

Tentin päivämäärä / Date of exam:21.1.2016	**Tentin kesto tunteina / Exam in hours:4**
Tiedekunta / Faculty: Oulu Business School	
Opintojakson koodi, nimi ja tentin numero / The code and the name of the course and number of the exam: 721338S, Mathematical Economics, 6 credits (3rd exam)	
Tentaattori(t) / Examiner(s): Matti Koivuranta	**Sisäinen postios. / Internal address** **Matti Koivuranta, Oulu Business School**
Sallitut apuvälineet / The devices allowed in the exam: ☒ Nelilaskin / Standard calculator ☐ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator ☐ Muu materiaali, tarkennettu alla / Other material, specified below:	
Tenttiin vastaaminen / Please answer the questions: ☒ Suomeksi / in Finnish ☒ Englanniksi / in English	
Kysymyspaperi on palautettava / Paper with exam questions must be returned: ☐ Kyllä / Yes ☒ Ei / No	

All five questions should be answered

MATHEMATICAL ECONOMICS
Exam 3, 21.1.2016
Matti Koivuranta

1. Assume following matrices:

$$A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -4 \\ 2 & -2 \end{bmatrix}.$$

   Calculate the following:

   a) sum $A + B$
   b) transpose B^T
   c) product AB
   d) determinant $|A|$
   e) inverse A^{-1}
   f) rank of matrix B

2. a) Solve equation $\frac{x}{4} = \sqrt{x}$.
   b) Differentiate function $f(x) = \ln\left(\frac{1}{x}\right)$.
   c) Compute definite integral $\int_{-2}^{2}(x^3 - x)dx$.

3. Let there be a set of equations

$$\begin{cases} x^2 + axy + y^2 = 1 \\ x^2 + y^2 + 3 = a^2. \end{cases}$$

   The equations hold when $x = 0$, $y = 1$ and $a = 2$. Do the equations determine x and y as functions of a around point $a = 2$. If yes, calculate the derivatives $\frac{dx}{da}$ and $\frac{dy}{da}$ at point $a = 2$.

4. Solve the utility maximization problem

$$\max_{\{x_1, x_2\}} \{\alpha \ln x_1 + \beta \ln x_2\} \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq w$$

   for the optimal consumption bundle $(x_1^*, x_2^*) \in \mathbb{R}^2_{++}$. Parameters α, β, p_1, p_2 and w are all strictly positive.

5. a) Find an explicit solution for difference equation $2y_{t+1} = y_t + 1$ with initial condition $y_0 = 1$.
   b) Find stationary points of differential equation $y'(t) = -y(t)^2 + 1$ and use graphical analysis to determine whether the stationary points are locally stable or not. Sketch the solution function $y(t)$ assuming initial condition $y(0) = 0$.

Following lemmas, which consider testing for local quality of critical points, may or may not be useful:

Lemma 1 (definiteness of Hessian) *Matrix A is*

1. *positive definite, if* $|A_i| > 0 \quad \forall i = 1, \ldots, n$.
2. *negative definite, if* $|A_1| < 0, |A_2| > 0, |A_3| < 0$, *etc. (sign alternates with first sign negative).*
3. *positive semi-definite, if* $|A_i| \geq 0 \quad \forall i = 1, \ldots, n$.
4. *negative semi-definite, if* $|A_1| \leq 0, |A_2| \geq 0, |A_3| \leq 0$, *etc.*
5. *indefinite, if none of the above holds.*

Lemma 2 (using bordered Hessian) *Assume an equality-constrained optimization problem with* n *choice variables and* m *constraints. Critical points of the Lagrangian can be classified by examining subdeterminants of the bordered Hessian.*

A critical point is a strict local minimum if the signs of the $n - m$ *largest subdeterminants are all equal to the sign of* $(-1)^m$. *A critical point is a strict local maximum if the signs of* $n - m$ *largest subdeterminants alternate and the sign of the largest subdeterminant is equal to the sign of* $(-1)^n$.