UNIVERSITY OF OULU

YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

Opintojakson koodi and nimi / The code and the name of the course:

721338S

Mathematical Economics

Tiedekunta / Faculty: Oulu Business School

Tentin pvm / Date of exam: 25.8.2017	**Tentin kesto tunteina / Exam in hours: 3**
Tentin nro / No. of the exam: 3rd retake	**Opintopistemäärä / Credit units: 6**
Tentaattori(t) / Examiner(s): Matti Koivuranta	**Sisäinen postios. / Internal address:** **Matti Koivuranta, Oulu Business School**

Sallitut apuvälineet / The devices allowed in the exam:

☒ Nelilaskin / Standard calculator

☐ Funktiolaskin / Scientific calculator

☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintojakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

☐ Kyllä / Yes ☒ Ei / No

All five questions shall be answered

MATHEMATICAL ECONOMICS
Exam, 25.8.2017
Matti Koivuranta

All five questions shall be answered
No own calculators allowed

1. Assume following matrices:

$$A = \begin{bmatrix} -4 & -1 \\ 4 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}.$$

   Calculate the following:

   a) sum $A + B$
   b) transpose B^T
   c) product AB
   d) determinant $|A|$
   e) inverse A^{-1}
   f) rank of matrix B

2. a) Solve equation $2x^{-\frac{1}{2}} = 1$.
   b) Differentiate function $f(x) = (x^2 - x + 2)e^{2x}$.
   c) Compute definite integral $\int_0^1 (x^2 + 1)dx$.

3. Equation $e^{y+x} = y$ holds when $x = 1 - e$ and $y = e$. Does the equation determine y as a function of x around point $x = 1 - e$? If yes, calculate the derivative $\frac{dy}{dx}$ at point $x = 1 - e$.

4. Solve the utility maximization problem

$$\max_{\{x_1, x_2\}} \{x_1^{\alpha} x_2^{1-\alpha}\}$$
$$\text{s.t.} \quad p_1 x_1 + p_2 x_2 \leq w$$
$$x_1, x_2 \geq 0$$

   for the optimal consumption bundle (x_1^*, x_2^*). Parameters satisfy $p_1, p_2, w > 0$ and $0 < \alpha < 1$.

5. a) Find an explicit solution for difference equation $4y_{t+1} = y_t + 6$ with initial condition $y_0 = 9$.
   b) Find stationary points of differential equation $y'(t) = -y(t)^3 + y(t)$ and use graphical analysis to determine whether the stationary points are locally stable or not. Sketch the solution function $y(t)$ assuming initial condition $y(0) = -\frac{1}{2}$.

Following lemmas, which consider testing for local quality of critical points, may or may not be useful:

Lemma 1 (definiteness of Hessian) *Matrix A is*

1. *positive definite, if* $|A_i| > 0 \quad \forall i = 1, \ldots, n$.
2. *negative definite, if* $|A_1| < 0, |A_2| > 0, |A_3| < 0$, *etc. (sign alternates with first sign negative).*
3. *positive semi-definite, if* $|A_i| \geq 0 \quad \forall i = 1, \ldots, n$.
4. *negative semi-definite, if* $|A_1| \leq 0, |A_2| \geq 0, |A_3| \leq 0$, *etc.*
5. *indefinite, if none of the above holds.*

Lemma 2 (using bordered Hessian) *Assume an equality-constrained optimization problem with* n *choice variables and* m *constraints. Critical points of the Lagrangian can be classified by examining subdeterminants of the bordered Hessian.*

A critical point is a strict local minimum if the signs of the $n - m$ *largest subdeterminants are all equal to the sign of* $(-1)^m$. *A critical point is a strict local maximum if the signs of* $n - m$ *largest subdeterminants alternate and the sign of the largest subdeterminant is equal to the sign of* $(-1)^n$.