

## YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:

Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

Opintjakson koodi and nimi / The code and the name of the course:

**721338S**

**Mathematical Economics**

Tiedekunta / Faculty: Oulun yliopiston kauppakorkeakoulu / Oulu Business School

Tentin pvm / Date of exam: 30.10.2017

Tentin kesto tunteina / Exam in hours: 3 h

Tentaattori(t) / Examiner(s):

Juha Teirilä

Opintopistemäärä / Credit units: 6

Sisäinen postios. / Internal address:

6 OyKKK

Sallitut apuvälineet / The devices allowed in the exam:

Funktiolaskin /  
Scientific calculator       Ohjelmoitava laskin /  
Programmable calculator

Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

Suomeksi / in Finnish       Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintjakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

Kyllä / Yes       Ei / No

Please answer all 5 questions (6 points each).

721338S MATHEMATICAL ECONOMICS  
1st final exam, 30.10.2017

1. Assume a matrix:

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -2 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}.$$

Calculate the following:

- a) transpose  $A^T$
- b) determinant  $|A|$
- c) rank of the matrix  $A$
- d) inverse matrix  $A^{-1}$
- e) definiteness of the matrix  $A$

2. What is the Hessian matrix? How is it formed and how it can be used when solving optimization problems? Use examples if needed.

3. One solution for the following system of equations:

$$\begin{cases} y^2 + 2u^2 + v^2 - xy = 15 \\ 2y^2 + u^2 + v^2 + xy = 38 \end{cases}$$

is a point  $(x = 1, y = 4, u = 1, v = -1)$ . Assume that variables  $x$  and  $y$  are endogenous while  $u$  and  $v$  are exogenous. Solve partial derivatives  $\frac{\partial x}{\partial u}$  and  $\frac{\partial y}{\partial v}$  in a given point.

4. Solve the critical points for the following maximization problem and form a matrix that can be used to classify them (no need to do the classification):

$$\max_{\{x,y,z\}} \{x + y + z^2\} \quad \text{s.t.} \quad x^2 + y^2 + z^2 = 1 \quad \text{and} \quad y = 0.$$

5. Assume following demand ( $Q_t^d$ ) - supply ( $Q_t^s$ ) model where time  $t$  is discrete and price is denoted with  $P$ :

$$\begin{cases} Q_t^d = Q_t^s \\ Q_t^d = 22 - 4P_t \\ Q_t^s = -3 + P_{t-1}. \end{cases}$$

- a) Solve price as a function of time ( $P_t$ ) when price is 8 at time period 0 ( $P_0 = 8$ ).
- b) Does the model have dynamically stable equilibrium? If yes, what is it? Draw a graph.