LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit)
SCI; OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

UNIVERSITY OF OULU

YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:

Opettaja täyttää / Lecturer fills in:

Opintojakson koodi and nimi / The code and the name of the course:	
721338S **Mathematical Economics**	
Tiedekunta / Faculty: Oulun yliopiston kauppakorkeakoulu / Oulu Business School	
Tentin pvm / Date of exam: 4.12.2017	Tentin kesto tunteina / Exam in hours: 3 h
Tentaattori(t) / Examiner(s): Juha Teirilä	Opintopistemäärä / Credit units: 6
	Sisäinen postios. / Internal address: 6 OyKKK

Sallitut apuvälineet / The devices allowed in the exam:

☐ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintojakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

☒ Kyllä / Yes ☐ Ei / No

Please answer all 5 questions (6 points each).

721338S MATHEMATICAL ECONOMICS
2nd final exam, 4.12.2017

1. Solve the following system of equations using Cramer's rule

$$\begin{cases} 4x + 3y = 1 + 2z \\ x + y = 6 - y \\ 3x + z - 4 = 0. \end{cases}$$

2. What is the *bordered* Hessian matrix? How is it formed and how it can be used when solving optimization problems?

3. Assume an equation

$$x^2 + z^2 + 3zy + 2yx + y^2 = 11.$$

We are interested in partial derivatives $\frac{\partial x}{\partial y}$ and $\frac{\partial x}{\partial z}$. What form does the implicit function then have? Does this implicit function exist in the neighborhood of point $(x = 0, y = 2, z = 1)$? If yes, solve the desired partial derivatives in this point.

4. Find such consumption bundle (c_1^*, c_2^*), which maximizes a utility function (of a two period model) $u(c_1, c_2) = \ln c_1 + \beta \ln c_2$ on the budget line $c_1 + \frac{c_2}{R} = I$. Here, $c_1 \geq 0$ is consumption at period 1, $c_2 \geq 0$ is consumption at period 2 and $I > 0$ is income. Gross interest rate $R > 0$ and time preference $0 < \beta < 1$ are model parameters. Explain why the found point is a maximum.

5. Let the demand $Q_d(t)$ and supply $Q_s(t)$ at a given time t be

$$\begin{cases} Q_d(t) = 20 - 4P(t) - 2\frac{dP}{dt} \\ Q_s(t) = 2P(t), \end{cases}$$

where $P(t)$ is price at time t.

   a) Assuming that the market is cleared at every point of time ($Q_d(t) = Q_s(t) \, \forall \, t$), find $P(t)$, the time path of the price.

   b) Does this market have a dynamically stable intertemporal equilibrium price?