UNIVERSITY OF OULU

YLEISEN TENTIN TENTTILOMAKE - GENERAL EXAM FORM

Opiskelija täyttää / Student fills in

Opiskelijan nimi / Student name:	Opiskelijanumero / Student number:
Click here to enter text.	Click here to enter text.

Opettaja täyttää / Lecturer fills in

Opintojakson koodi / The code of the course:
721338S

Opintojakson (tentin) nimi / The name of the course or exam:
Mathematical Economics

Opintopistemäärä / Credit units:
6 cr

Mikäli kyseessä on välikoe, opintopistemääräksi täytetään 0 op.
0 ECTS Credits is used for mid-term exams.

Tiedekunta / Faculty: Oulu Business School

Tentin pvm / Date of exam:	Tentin kesto tunteina / Exam in hours: 3 h
29.10.2018	
Tentaattori(t) / Examiner(s):	**Sisäinen postiosoite / Internal address:**
Tomi Alaste	TA331

Tentissä sallitut apuvälineet / The devices allowed in the exam:

☐ Funktiolaskin / Scientific calculator

☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu tentissä sallittu materiaali tai apuvälineet. Tarkenna alla. / Other material or devices, allowed in the exam. Specify below.

Click here to enter text.

☒ Tentissä ei ole sallittua käyttää apuvälineitä / The devices are not allowed in the exam

Muut tenttiä koskevat ohjeet opiskelijalle (esimerkiksi kuinka moneen kysymyksen opiskelijan tulee vastata) / Other instructions for students e.g. how many questions he/she should answer:

Answer all the questions.

721338S MATHEMATICAL ECONOMICS
Final exam
29.10.2018

1. Consider the following system of equations:

$$\begin{cases} x + z = 0 \\ y + z = 1 \\ -x + 2y = 1 \end{cases}$$

(a) Write this system in matrix form $Ax = b$. (5 p.)

(b) Calculate $\det A$. (5 p.)

(c) Which of the following is the inverse matrix of A? (5 p.)

$$B = \begin{bmatrix} 0 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix}.$$

(d) Solve x, y, and z. (5 p.)

2. Suppose that the demand and the supply of some good are given by

$$\begin{cases} Q_d = a - bP + cI, \\ Q_s = -d + eP, \end{cases}$$

where P is price, I is income, and $a, b, c, d, e > 0$ are parameters.

(a) Solve the equilibrium quantity Q^* and price P^*. (10 p.)

(b) What happens to the equilibrium quantity and price when income increases? (10 p.)

3. Consider the following utility function of two goods, where $a, b > 0$ are parameters:

$$U(x, y) = a \ln x + b \ln y.$$

Let $p_x > 0$ and $p_y > 0$ be the prices of the goods and $I > 0$ the available income. The purpose is to find the utility maximizing bundle.

(a) What is the constraint in this problem? (5 p.)

(b) Find the utility maximizing bundle. (*Hint: first order conditions are sufficient for global maximum.*) (10 p.)

(c) In general, what would you use to classify the critical points in an unconstrained optimization problem and in a constrained optimization problem? (5 p.)

4. Consider the following system of equations:

$$\begin{cases} xy = w, \\ y - 3z = w^3, \\ z^3 - 2zw + w^3 = 0. \end{cases}$$

These equations are satisfied when $x = 1/4$, $y = 4$, $w = 1$, and $z = 1$. (*You do not need to verify this.*)

(a) Show that x, y, and w are functions of z in some neighborhood of the given point. (10 p.)

(b) Give an equation which can be used to solve for $\frac{dx}{dz}$, $\frac{dy}{dz}$, and $\frac{dw}{dz}$. (5 p.)

(c) Solve $\frac{dx}{dz}$. (5 p.)

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & 0 \end{vmatrix} \quad \begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ -1 & 2 & -1 \end{bmatrix} =$$

2 0 -1 1

0 1 -1 -1 0 -1 2

-1 2 0

$-3(1)^2 + 2$

-1

$2z - 3w^2$

$2(1) - 3(1)^2$

$2 - 3 = -1$

5. (a) Solve the differential equation $y'(t) = -3t + 2$. (5 p.)

(b) Consider a Cobb-Douglas production function $Y_t = K_t^{\theta}(A_t L_t)^{1-\theta}$, where $0 < \theta < 1$. The law of motion of capital K is given by

$$K_{t+1} = (1 - \delta)K_t + I_t,$$

where $0 < \delta < 1$. Suppose that investments are a constant share, $0 < s < 1$, of the output, that is, $I_t = sY_t$. Labor productivity A and labor L grow at constant rates according to $A_{t+1} = \gamma A_t$ and $L_{t+1} = nL_t$ where $n, \gamma \geq 1$. Solve for the law of motion of capital per effective labor $k = \frac{K}{AL}$. Solve also the stationary points of k and determine whether they are stable or not. (15 p.)

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & 0 \end{vmatrix} \circ \begin{vmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ -1 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & -1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{vmatrix}$$

$\begin{matrix} -2 & 0 & 2 \\ 0 & -1 & 2 \\ -2 & -2 & 0 \end{matrix}$ $\begin{matrix} 1 & 0 & -1 \\ -0 & 1 & -1 \\ -1 & 2 & 0 \end{matrix}$

$\begin{matrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

$2 \times \beta = -3$

$\beta = \frac{-3}{2}$