UNIVERSITY OF OULU

YLEISEN TENTIN TENTTILOMAKE - GENERAL EXAM FORM

Opiskelija täyttää / Student fills in

Opiskelijan nimi / Student name:	**Opiskelijanumero / Student number:**
Click here to enter text.	Click here to enter text.

Opettaja täyttää / Lecturer fills in

Opintojakson koodi / The code of the course: 721338S	
Opintojakson (tentin) nimi / The name of the course or exam: Mathematical Economics	
Opintopistemäärä / Credit units: 6 cr Mikäli kyseessä on välikoe, opintopistemääräksi täytetään 0 op. 0 ECTS Credits is used for mid-term exams.	
Tiedekunta / Faculty: Oulu Business School	
Tentin pvm / Date of exam: 3.12.2018	**Tentin kesto tunteina / Exam in hours: 3 h**
Tentaattori(t) / Examiner(s): Tomi Alaste	**Sisäinen postiosoite / Internal address:** 6OyKKK
Tentissä sallitut apuvälineet / The devices allowed in the exam:	
☐ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator ☐ Muu tentissä sallittu materiaali tai apuvälineet. Tarkenna alla. / Other material or devices, allowed in the exam. Specify below. Click here to enter text. ☒ Tentissä ei ole sallittua käyttää apuvälineitä / The devices are not allowed in the exam	
Muut tenttiä koskevat ohjeet opiskelijalle (esimerkiksi kuinka moneen kysymyksen opiskelijan tulee vastata) / Other instructions for students e.g. how many questions he/she should answer: Answer all the questions.	

721338S MATHEMATICAL ECONOMICS
Final exam
3.12.2018

1. Consider the following system of equations:

$$\begin{cases} x + z = 3 \\ y + z = 2 \\ 2x + y = 2 \end{cases}$$

   (a) Write this system in the matrix form $Ax = b$. (5 p.)

   (b) Calculate $\det A$. (5 p.)

   (c) Is A invertible, that is, does A^{-1} exist? (5 p.)

   (d) Solve y. (5 p.)

2. Suppose that the demand and the supply of some good are given by

$$\begin{cases} Q_d = a - bP + cI, \\ Q_s = -d + eP, \end{cases}$$

   where P is price, I is income, and $a, b, c, d, e > 0$ are parameters.

   (a) Find the equilibrium quantity Q^* and price P^*. (10 p.)

   (b) What happens to the equilibrium price when the income *decreases*? (5 p.)

   (c) Find the income-elasticity of demand. (5 p.)

3. Consider the following utility function of two goods, where $0 < \alpha < 1$:

$$U(x, y) = x^\alpha y^{1-\alpha}.$$

   Let $p_x > 0$ and $p_y > 0$ be the prices of the goods and $I > 0$ the available income. Suppose that a consumer spends all his/her income, so that the constraint is given by $p_x x + p_y y = I$.

   (a) Form the Lagrangian. (5 p.)

   (b) Find the utility maximizing bundle (x^*, y^*). (*Hint: first order conditions are sufficient for the maximum.*) (5 p.)

   (c) Put $U^* = U(x^*, y^*)$, that is, U^* is the achieved utility when the optimal bundle (x^*, y^*) is chosen. Note that this depends on I. Find $\frac{\partial U^*}{\partial I}$ and explain in words what this derivative represents. (10 p.)

4. Consider the following national-income model

$$\begin{cases} Y - C - I_0 - G_0 = 0, \\ C - \alpha - \beta(Y - T) = 0, \\ T - \gamma - \delta Y = 0, \end{cases}$$

   where $\alpha, \gamma > 0$ and $0 < \beta, \delta < 1$. Suppose that we have an equilibrium, that is, the above equations are satisfied.

   (a) Show that Y, C, and T are functions of I_0, G_0, α, β, γ, and δ in some neighbourhood of the equilibrium point. (10 p.)

   (b) Solve $\frac{\partial Y}{\partial G_0}$ and $\frac{\partial C}{\partial I_0}$. (10 p.)

5. (a) Solve the differential equation $4y = 2y'$ with the initial condition $y(0) = 1$. (10 p.)

   (b) Consider the following discrete time model of supply (Q_t^s) and demand (Q_t^d), where P_t denotes the price at time t:

$$\begin{cases} Q_t^d = Q_t^s, \\ Q_t^d = 20 - 4P_t, \\ Q_t^s = -4 + 2P_{t-1}. \end{cases}$$

   Solve the price P_t as a function of t when $P_0 = 8$. (10 p.)