UNIVERSITY OF OULU

YLEISEN TENTIN TENTTILOMAKE - GENERAL EXAM FORM

Opiskelija täyttää / Student fills in

Opiskelijan nimi / Student name:	**Opiskelijanumero / Student number:**
Click here to enter text.	Click here to enter text.

Opettaja täyttää / Lecturer fills in

Opintojakson koodi / The code of the course:	
721338S	
Opintojakson (tentin) nimi / The name of the course or exam:	
Mathematical Economics	
Opintopistemäärä / Credit units:	
6 cr Mikäli kyseessä on välikoe, opintopistemääräksi täytetään 0 op. 0 ECTS Credits is used for mid-term exams.	
Tiedekunta / Faculty: Oulu Business School	
Tentin pvm / Date of exam:	**Tentin kesto tunteina / Exam in hours: 3 h**
16.1.2019	
Tentaattori(t) / Examiner(s):	**Sisäinen postiosoite / Internal address:**
Tomi Alaste	6OyKKK
Tentissä sallitut apuvälineet / The devices allowed in the exam:	
☐ Funktiolaskin / Scientific calculator ☐ Ohjelmoitava laskin / Programmable calculator ☐ Muu tentissä sallittu materiaali tai apuvälineet. Tarkenna alla. / Other material or devices, allowed in the exam. Specify below. Click here to enter text. ☒ Tentissä ei ole sallittua käyttää apuvälineitä / The devices are not allowed in the exam	
Muut tenttiä koskevat ohjeet opiskelijalle (esimerkiksi kuinka moneen kysymyksen opiskelijan tulee vastata) / Other instructions for students e.g. how many questions he/she should answer: Answer all the questions.	

721338S MATHEMATICAL ECONOMICS
Final exam
16.1.2019

1. Consider the following system of equations:

$$\begin{cases} x + y = 0 \\ 2x + y = 1 \end{cases}$$

   (a) Write this system in matrix form $Ax = b$. (5 p.)

   (b) Find the inverse matrix of A and check that the matrix you have found really is the inverse matrix of A. (10 p.)

   (c) Solve x and y. (5 p.)

2. Consider the following simple national-income model

$$\begin{cases} Y = C + I_0 + G_0, \\ C = a + bY, \end{cases}$$

   where Y is national income, C is consumption, I_0 is investment, G_0 is government expenditure, and $a > 0$ and $0 < b < 1$ are parameters.

   (a) Solve Y and C in terms of I_0, G_0, a, and b. (10 p.)

   (b) What happens to the national income Y when investment I_0 *increases*? (5 p.)

   (c) What happens to the consumption C when govenrment expenditure G_0 *decreases*? (5 p.)

3. The purpose of this exercise is to minimize the function

$$f(x, y) = x - 3y - xy$$

   under the constraint $x + y = 6$.

   (a) Form the Lagrangian. (5 p.)

   (b) Find the critical points. (5 p.)

   (c) Form the bordered Hessian at the critical point. (*There is no need to try to qualify the critical point.*) (5 p.)

   (d) Explain shortly in words what is the role of Hessian matrices and bordered Hessians matrices in optimization problems. (5 p.)

4. Let $x = 0$, $y = 1$, and $a = 2$ and consider the following equations:

$$\begin{cases} x^2 + axy + y^2 = 1, \\ x^2 + y^2 + 3 = a^2, \end{cases}$$

   (a) Show that x and y are functions of a in some neighbourhood of the given point. (10 p.)

   (b) Find $\frac{dx}{da}$ and $\frac{dy}{da}$ at the given point. (10 p.)

5. (a) Solve the differential equation $y'(t) = 4t$ with the initial condition $y(0) = 1$. (10 p.)

   (b) Consider the following discrete time model of supply (Q_t^s) and demand (Q_t^d), where P_t denotes the price at time t:

$$\begin{cases} Q_t^d = Q_t^s, \\ Q_t^d = 20 - 4P_t, \\ Q_t^s = -4 + 2P_{t-1}. \end{cases}$$

   Solve the price P_t as a function of t when $P_0 = 4$. (10 p.)