LuTK, OyKKK, KaTK, TTK, TST ja BMTK (Linnanmaan tentit)
SCI, OBS, OMS, TECH, ITEE and BMM (Linnanmaa campus)

UNIVERSITY OF OULU

YLIOPISTOTENTTI - UNIVERSITY EXAM

Opiskelijan nimi / Student name:	**Opiskelijanumero / Student number:**

Opettaja täyttää / Lecturer fills in:

Opintojakson koodi and nimi / The code and the name of the course:

721338S

Mathematical Economics

Tiedekunta / Faculty: Oulun yliopiston kauppakorkeakoulu / Oulu Business School

Tentin pvm / Date of exam: 15.1.2018	**Tentin kesto tunteina / Exam in hours: 3 h**
Tentaattori(t) / Examiner(s):	**Opintopistemäärä / Credit units: 6**
Juha Teirilä	**Sisäinen postios. / Internal address:**
	6 OyKKK

Sallitut apuvälineet / The devices allowed in the exam:

☐ Funktiolaskin / Scientific calculator

☐ Ohjelmoitava laskin / Programmable calculator

☐ Muu materiaali, tarkennettu alla / Other material, specified below:

Tenttiin vastaaminen / Please answer the questions:

☒ Suomeksi / in Finnish ☒ Englanniksi / in English

Suomenkielisessä tutkinto-ohjelmassa olevalla opiskelijalla on oikeus käyttää arvioitavassa opintosuorituksessa suomen kieltä, vaikka opintojakson opetuskieli olisi englanti. Tämä ei koske vieraan kielen opintoja. (Kts. Koulutuksen johtosääntö 18 §)

In a Finnish degree programme a student has a right to use Finnish language for their study attainment, even though the language of instruction is English, (excluding language studies) even when the language of instruction is other than Finnish. (See the Education Regulations 18 §)

Kysymyspaperi on palautettava / Paper with exam questions must be returned:

☒ Kyllä / Yes ☐ Ei / No

Please answer all 5 questions (6 points each).

721338S MATHEMATICAL ECONOMICS
3rd final exam, 15.1.2018

1. Solve the following system of equations using a matrix inversion (i.e. form a matrix equation and solve it)

$$\begin{cases} -x + y - z = 8 \\ 4x + y - 8z = 5 \\ 6x - 4y + 5z = 1. \end{cases}$$

2. a) What is the value of the Hessian matrix of a function $g(x, y)$ at a point $(x = 1, y = 0)$, when the function $g : \mathbb{R}^2 \to \mathbb{R}$ is

$$g(x, y) = 2x^2 + 4xy + 3xy^2 - e^{2y}.$$

b) Solve the following definite integral

$$\int_{-1}^{1} (5x^4 - 6x^2 + 2)\, dx.$$

3. The following system of equations

$$\begin{cases} y^2 - xy + 2z^2 = 14 \\ 2y^2 + xy + z^2 = 37 \end{cases}$$

has a solution at a point $(x, y, z) = (1, 4, 1)$. Show that the system of equations defines variables x and y as functions of variable z in the neighborhood of a point $z = 1$. Solve the partial derivatives of these implicitly defined functions at the point $z = 1$.

4. Find such consumption bundle (c_1, c_2), which maximizes a utility function

$$U(c_1, c_2) = a \ln c_1 + b \ln c_2$$

subject to budget constraint $p_1 c_1 + p_2 c_2 = I$. Here c_1 and c_2 are quantities of good 1 and 2, $p_1 > 0$ and $p_2 > 0$ are prices, $I > 0$ is the available income and $a, b > 0$ are parameters of the utility function. (Hint: the given utility function is strictly quasi-concave and the constraint is linear. Therefore the first order conditions are sufficient for a global maximum.)

5. Solve the differential equation

$$2y'(t) + 3y(t) = 4$$

with an initial condition $y(0) = 2$. What is the limit of the solution at infinity?