

YLEISEN TENTIN TENTTILOMAKE - GENERAL EXAM FORM

Opiskelija täyttää / Student fills in

Opiskelijan nimi / Student name: Click here to enter text.	Opiskelijanumero / Student number: Click here to enter text.
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Opettaja täyttää / Lecturer fills in

Opintojakson koodi / The code of the course: 721338S	
Opintojakson (tentin) nimi / The name of the course or exam: Mathematical Economics	
Opintopistemäärä / Credit units: 6 cr	
Mikäli kyseessä on välikoe, opintopistemääräksi täytetään 0 op. 0 ECTS Credits is used for mid-term exams.	
Tiedekunta / Faculty: Oulu Business School	
Tentin pvm / Date of exam: 11.12.2019	Tentin kesto tunteina / Exam in hours: 3 h
Tentaattori(t) / Examiner(s): Tomi Alaste	Sisäinen postiosoite / Internal address: 6OyKKK
Tentissä sallitut apuvälineet / The devices allowed in the exam:	
<input type="checkbox"/> Funktiolaskin / Scientific calculator <input type="checkbox"/> Ohjelmoitava laskin / Programmable calculator <input type="checkbox"/> Muu tentissä sallittu materiaali tai apuvälineet. Tarkenna alla. / Other material or devices, allowed in the exam. Specify below. Click here to enter text. <input checked="" type="checkbox"/> Tentissä ei ole sallittua käyttää apuvälineitä / The devices are not allowed in the exam	
Muut tenttiä koskevat ohjeet opiskelijalle (esimerkiksi kuinka moneen kysymyksen opiskelijan tulee vastata) / Other instructions for students e.g. how many questions he/she should answer: Answer all the questions.	

1. Let us study the following system of equations:

$$\begin{cases} x + y = 0, \\ 2x + y = 1. \end{cases}$$

- (a) Write this system in matrix form $Ax = b$.
(b) Calculate $\det A$.
(c) Is A invertible? If so, find the inverse matrix A^{-1} .
(d) Solve x and y .
2. Find the critical points of the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = x^4 + x^2 - 6xy + 3y^2$. Check also the quality of critical points.

3. Suppose there are two goods available, and a consumer's utility with amounts x and y of these goods is given by

$$U(x, y) = xy.$$

The consumer has income I and the prices of the goods are p_x and p_y , respectively. The purpose is to find a utility maximizing bundle (x^*, y^*) .

- (a) Assuming that the consumer spends all the income, what is the constraint in this problem?
(b) Find the optimal bundle by forming the Lagrangian and finding the critical point(s).
(c) Find the bordered Hessian. (It is enough to give this matrix; there is no need to qualify the critical point.)
4. Let us study the following set of equations:

$$\begin{cases} xz^3 + y^2v^4 = 2 \\ xz + yvz^2 = 2. \end{cases}$$

These equations are satisfied when $x = y = z = v = 1$.

- (a) Show that z and v are functions of x and y near the given point.
(b) Solve the partial derivatives $\partial z / \partial x$ and $\partial v / \partial y$.
5. (a) Solve the differential equation $y'(t) = 4t + 2$ under the initial condition $y(0) = 1$. (**Hint:** If you don't remember how to solve this, you may substitute $y(t) = at^2 + bt + c$ and find the coefficients a , b , and c .)
(b) Solve the differential equation $y' - 2y = 1$ under the initial condition $y(0) = 1$.