Taloustieteiden tiedekunta

Tentin päivämäärä / Date of exam: 05.06.2015
Opintojakson koodi, nimi ja tentin numero / The code and the name of the course and number of the exam: **724208A Portfolio Theory (summer exam)**
Tentaattori(t)/ Examiner(s): Andrew Conlin
Sallitut apuvälineet / The devices allowed in the exam: X Laskin (ei graafinen/ohjelmoitava)/Calculator (not graphic, programmable) ☐ Sanakirja/Dictionary ☐ Muu materiaali, tarkennettu alla/Other material, specified below
Tenttiin vastaaminen / Please answer the questions X suomeksi/ in Finnish X englanniksi/ in English
Kysymyspaperi on palautettava / Paper with exam questions must be returned: ☐ Kyllä/Yes X Ei/No

The exam consists of 10 quantitative questions (2 points each) and 5 short-answer essay questions (4 points each). There is a formula sheet attached. **WRITE CLEARLY and SHOW YOUR WORK** (no credit for answers if you do not show your work!) You may answer in English or Finnish. If you answer in Finnish, use yleiskieli.

Part 1

Questions 1-3 use the following information. You own shares of Stock A and Stock B in your portfolio. The total value of your Stock A shares is 1500€. The total value of your Stock B shares is 1000€. The table shows the different states of the economy, the probability, and the return for each stock, for the upcoming year.

State of the economy	Probability	r_A	r_B
Boom	0.25	12%	25%
Normal	0.60	8%	8%
Bust	0.15	-5%	-30%

1. What is the expected return on Stock A?
2. What is the expected return on your portfolio?
3. What is the standard deviation of your portfolio?

4. Assume 1 risky asset and 1 risk-free asset. Also assume the utility function $U = E(r) - ½ A \sigma^2$. You have risk aversion = 2.5. The risky asset has $E(r_p)$= 10% and σ_p =.22 The risk-free rate is 1.5%. How much of your portfolio should you invest in the risky asset?

Questions 5-7 are related.

Assume a world with a risky debt portfolio, a risky equity portfolio, and a risk-free asset. The debt portfolio has $E(r_d)$ = 4 % and σ_d = 11%. The equity portfolio has $E(r_e)$=12% and σ_e = 24%. The risk-free rate is 2%. The <u>correlation</u> between the debt and equity portfolios is $Corr(r_d, r_e)$ = 0.095

5. What are the weights of the debt and equity portfolios in the minimum variance portfolio?
6. What is the expected return on the optimal risky portfolio?
7. If you have risk aversion = 3.5, how much of your money should you put in the optimal risky portfolio (assume the utility function $U = E(r) - ½ A \sigma^2$)?

Questions 8 and 9 are related.

You sold short 1000 shares of Nordea at a price of 12.00. The initial margin is 50%. You earn no interest on the funds in your account and the cost of borrowing the shares is 0.25%.

8. If the maintenance margin is 35%, how high can the stock go before you get a margin call?
9. Nordea paid a dividend of 0.6€ per share while you held the short position. You covered the short position at a price of 11.75. What was your return on the short position?

10. Assume the index model. The risk-free rate is 1%. The market portfolio has $E(r_M)$ = 9% and σ_M= 22%. The active portfolio has α_A = 2.5% and $\sigma_{e_A} = 30\%$. What is the Sharpe ratio of the optimal risky portfolio?

<u>Part 2</u> Write maximum 2 paragraphs (tekstikappalet) per question. Write complete sentences; do NOT just list vocabulary words.

1. Define the efficient frontier and explain its role in optimal portfolio construction.
2. What is the disposition effect, and why is it bad behavior for investors?
3. Assume returns can be explained by a two-factor model (i.e. Arbitrage Pricing Theory). Portfolio X has β_{GDP} = 1.1 and β_{INF} = 0.2. Portfolio X is priced so it has E(r) = 14%. If but is priced to provide a return different than the return predicted by Arbitrage Pricing Theory, describe how you would form the tracking portfolio for portfolio X (i.e. what should you invest in and with what weights), and explain *why* this creates an arbitrage opportunity.
4. If the stock market is truly semi-strong form efficient, what does this imply for small investors who try to beat the market by investing in a few individual stocks?
5. Why is the concept "limits to arbitrage" a core part of behavioral finance? Give (at least) one specific example of limited arbitrage.

Formula Sheet - ***Kaavakokoelma***

$$(1+R)=(1+r)(1+i)$$

$$APR=\frac{(1+EAR)^{T}-1}{T}$$

$$\sigma^{2} = \sum_{s} p(s)[r(s) - E(r)]^{2}, \quad \sigma = \sqrt{\sigma^{2}}$$

$$Cov(r_i, r_j) = \sum_{s} p(s)[r_i(s) - E(r_i)][r_j(s) - E(r_j)]$$

$$S = \frac{E(r_P) - r_f}{\sigma_P}$$

$$y^{*} = \frac{E(r_P) - r_f}{A\sigma_P^{2}}$$

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$

$$\sigma_p^{2} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

$$Cov(r_i, r_j) = \rho_{ij}\sigma_i\sigma_j$$

$$w_{A_{Min}} = \frac{\sigma_B^{2} - Cov(r_A, r_B)}{\sigma_A^{2} + \sigma_B^{2} - 2Cov(r_A, r_B)}; w_{B_{Min}} = 1 - w_{A_{Min}}$$

$$w_A = \frac{[E(r_A)-r_f]\sigma_B^{2} - [E(r_B)-r_f]Cov(r_A,r_B)}{[E(r_A)-r_f]\sigma_B^{2} + [E(r_B)-r_f]\sigma_A^{2} - [E(r_A)-r_f + E(r_B)-r_f]Cov(r_A,r_B)}; w_B = 1 - w_A$$

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

$$\sigma_{i_{or}P}^{2} = \beta_{i_{or}P}^{2}\sigma_M^{2} + \sigma^{2}(e_{i_{or}P})$$

$$Cov(r_i, r_j) = \beta_i\beta_j\sigma_M^{2}$$

$$\alpha_P = \sum_{i=1}^{n} w_i \alpha_i; \quad \beta_P = \sum_{i=1}^{n} w_i \beta_i; \quad \sigma^2(e_P) = \sum w_i^2 \sigma^2(e_i)$$

$$w_i^0 = \frac{\alpha_i}{\sigma^2(e_i)} \quad \rightarrow \quad w_i = \frac{w_i^0}{\sum_{i=1}^{n} w_i^0}$$

$$w_A^0 = \frac{\alpha_A / \sigma^2(e_A)}{E(R_M) / \sigma_M^2} \quad \rightarrow \quad w_A^* = \frac{w_A^0}{1 + (1 - \beta_A) w_A^0}$$

$$S_P^2 = S_M^2 + \left[\frac{\alpha_A}{\sigma(e_A)} \right]^2$$

$$E(r_i) = r_f + \beta_i \left[E(r_M) - r_f \right]$$

$$\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}$$

$$E(r_P) = r_f + \beta_{P1} \left[E(r_1) - r_f \right] + \beta_{P2} \left[E(r_2) - r_f \right] + \ldots$$